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Approximate Solution of a Large Fractional Order Ebola Virus Disease Model with Control Measures via the Laplace - Adomian Decomposition Method

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Abstract

In this paper we presented an Ebola Virus Disease fractional order model in the Caputo sense with eighteen compartments. This model considered vaccination, condom use, quarantine, use of treatment drugs and isolation as control measures combined together. We showed that the model has non-negative solution. The stability of the equilibria was discussed and Laplace Adomian Decomposition Method is used to obtain the analytic or approximate solution of the system of non-linear fractional differential equation. It was observed or shown that with the Laplace Adomian Decomposition Method an approximate solution of a fractional order Ebola Virus Disease Model as large as eighteen compartments can be obtained and the result is close to the exact solution.

Keywords: Laplace Adomian Decomposition Method; Ebola Virus Disease; Fractional Order Model; Analytic solutions; Approximate Solutions

Introduction

Ebola Virus Disease (EVD) also referred to as Ebola hemorrhagic fever is actually a disease that is deadly in nature and was discovered in the year 1976 near the Ebola River in Democratic Republic of Congo [14]. This deadly disease can affect or infect both human non - human pirates. Five of its species have been identified as Zaire Ebola Virus, Reston Ebola Virus, Sudan Ebola Virus, Bundibugyo Ebola Virus and Tai Forest Ebola Virus [20]. Since the disease discovery there has been so many outbreaks of the disease in Africa of which the 2014 Ebola outbreak in West Africa was termed the largest in the history of the disease with multiple countries affected. Research shows that arthropods, rodents and bats could be the host of the Ebola Virus and direct transmission from reservoirs or secondary infected animals may occur [17]. Human beings can get infected with Ebola Virus through direct contact with blood or body fluids of an Ebola Virus sick or infectious person, dead person and infected object and surfaces. Symptoms from the disease may ap-

pear anywhere between two to twenty-one days after infection [14, 24]. Some treatment drugs and vaccine drugs for the disease have been put to use against the virus and those that recover from the disease are far more than those that die from the virus. Vaccine drugs such as rVSV- EBOV is 70% to 100% effective to protect against the virus [18, 19]. Some flare up cases has been detected assuming sexual transmission because of persistent residual virus in the sperm of the disease recovered men [15, 16].

Fractional Calculus has so many applications in applied sciences hence in applied mathematics such as mathematical modelling. Some studies before now have shown that fractional calculus can describe rules and develop process of some natural science phenomena [1, 25]. It has also been found that differential systems in fractional-order have the advantage of clear parameter meaning, sample modelling and accurate description of some materials and processes with memory and genetic characteristics [3]. These days, there is a high growing interest within the circle of researchers to look into the role of fractional calculus in the area of modelling real- world problems such as Ebola Virus Disease outbreaks especially in Africa. In this research paper, a fractional - order Ebola Virus Disease epidemic model that incorporates vaccine, use of condom, quarantine, treatment drugs and isolation is proposed and analyzed. So many mathematical models have been proposed before now to explain, predict and quantify how effective different Ebola Virus Disease intervention since the inception of the disease and so many Ebola outbreaks in Africa [4-13]. All these studies we have looked into via this study have in so many ways produced good result and improved the disease existence but is limited in that in most of the studies the authors involved were making use of only the integer - order models. Those that based their own upon fractional calculus did not consider these resent interventions of control measures included in the fractional - order model used herein in this study [2, 11-13]. The fractional - order Ebola Virus Disease presented in this work is solved analytically using the Laplace Adomian decomposition Method. The Laplace Adomian decomposition Method is a wonderful combination of the Adomian Decomposition Method and the Laplace Transforms. The Adomian decomposition Method was introduced by Adomian and has been applied in solving so many problems in sciences and applied sciences since then. This method can be used for a system of Linear and Nonlinear ordinary and partial differential equations of classical order as well as the fractional order. The method provides the solution in a rapid convergent series with computable terms. In this method, perturbation or linearization is not required. It has no need of pre-defined step size and does not depend on parameter like needed for Homotopy Perturbation Method (HPM) and Homotopy Analysis Method (HAM).

Methods

Fractional order EVD model

The classical EVD model used to develop the fractional EVD model used here is given in [25, 1]. The developed fractional order EVD model in the Caputo sense is as follows;

$$\left.\begin{aligned}
&(1)\, D^{\alpha}S(t) = P - (\mu + r + K_1X)S + \tau R\\
&(2)\, D^{\alpha}S_v(t) = rS - (\mu + \lambda + K_2Z)S_v\\
&(3)\, D^{\alpha}S_u(t) = K_1XS - (\mu + \sigma + K_3Y)S_u\\
&(4)\, D^{\alpha}S_{vc}(t) = \lambda S_v - \left[\mu S_{vc} + \tfrac{1}{N}((m_1I_NS_{vc} + m_2I_TS_{vc} + m_3I_rS_{vc} + m_4D_uS_{vc})\right]\\
&(5)\, D^{\alpha}S_{vn}(t) = K_2ZS_v - \left[\mu S_{vn} + \tfrac{1}{N}((n_1I_NS_{vn} + n_2I_TS_{vn} + n_3I_rS_{vn} + n_4D_uS_{vn})\right]\\
&(6)\, D^{\alpha}S_{uc}(t) = \sigma S_u - \left[\mu S_{uc} + \tfrac{1}{N}((e_1I_NS_{uc} + e_2I_TS_{uc} + e_3I_rS_{uc} + e_4D_uS_{uc})\right]\\
&(7)\, D^{\alpha}S_{un}(t) = K_3YS_u - \left[\mu S_{un} + \tfrac{1}{N}((t_1I_NS_{un} + t_2I_TS_{un} + t_3I_rS_{un} + t_4D_uS_{un})\right]\\
&(8)\, D^{\alpha}E(t) = \tfrac{1}{N}[m_1I_NS_{vc} + m_2I_TS_{vc} + m_3I_rS_{vc} + m_4D_uS_{vc} +\\
&n_1I_NS_{vn} + n_2I_TS_{vn} + n_3I_rS_{vn} + n_4D_uS_{vn} + e_1I_NS_{uc} + e_2I_TS_{uc} + e_3I_rS_{uc} + e_4D_uS_{uc} +\\
&t_1I_NS_{un} + t_2I_TS_{un} + t_3I_rS_{un} + t_4D_uS_{un}] - (\mu + \alpha_1 + \alpha_2)E\\
&(9)\, D^{\alpha}E_Q(t) = \alpha_1E - (\mu + \theta_1)E_Q\\
&(10)\, D^{\alpha}E_T(t) = \alpha_2E - (\mu + \theta_2 + \rho)E_T\\
&(11)\, D^{\alpha}I_T(t) = C_1E_Q + C_2E_T - (d_1 + J_2)I_T\\
&(12)\, D^{\alpha}I_i(t) = C_3E_Q + C_4E_T - (d_2 + J_1)I_i\\
&(13)\, D^{\alpha}I_N(t) = C_5E_Q + C_6E_T - d_3I_N\\
&(14)\, D^{\alpha}R(t) = J_1I_i + J_2I_T + \rho E_T - \tau R\\
&(15)\, D^{\alpha}D_u(t) = d_3I_N + d_2I_i + d_1I_T\\
&(16)\, D^{\alpha}S_r(t) = \Lambda - \mu_rS_r\\
&(17)\, D^{\alpha}E_r(t) = \omega S_r - \mu_rE_r\\
&(18)\, D^{\alpha}I_r(t) = \phi E_r - d_4I_r
\end{aligned}\right\} \quad (1)$$

With initial conditions as;

$$\left.\begin{array}{c} S(0)=g_1, S_v(0)=g_2, S_u(0)=g_3\ S_{vc}(0)=g_4, S_{vn}(0)=g_5, S_{uc}(0)=g_6, \\ S_{un}(0)=g_7, E(0)=g_8, E_Q(0)=g_9, E_T(0)=g_{10}, I_T(0)=g_{11}, I_i(0)=g_{12}, \\ I_N(0)=g_{13}, R(0)=g_{14}, D_u(0)=g_{15}, S_r(0)=g_{16}, E_r(0)=g_{17}\ and\ I_r(0)=g_{18}. \end{array}\right\} \quad (2)$$

Where $S(t)$ represents the susceptible human population, $S_v(t)$ is the vaccinated human population, $S_u(t)$ is the unvaccinated human population, $S_{vc}(t)$ is the vaccinated condom using population, $S_{vn}(t)$ is the vaccinated non using condom population, $S_{uc}(t)$ is the unvaccinated condom using population, $S_{un}(t)$ is the unvaccinated non using condom population, $E(t)$ is the exposed human population, $E_Q(t)$ is the exposed quarantined human population, $E_T(t)$ is the exposed treated human population, $I_T(t)$ is the infectious treated human population, $I_i(t)$ is the infectious isolated human population, $I_N(t)$ is the infectious not treated human population, $R(t)$ is the recovered human population, $D_u(t)$ is the dead unburied human population, $S_r(t)$ is the susceptible animal population, $E_r(t)$ is the exposed animal population and $I_r(t)$ is the infectious animal population.

Equilibrium points and Stability of the Model

Here, we analyzed the dynamic system (1) qualitatively for feasibility at disease free and endemic equilibrium point.

At equilibrium;

$$\left.\begin{array}{c} D^\alpha S = D^\alpha S_v = D^\alpha S_u = D^\alpha S_{vc} = D^\alpha S_{vn} = D^\alpha S_{uc} = D^\alpha S_{un} = D^\alpha E = D^\alpha E_Q = D^\alpha E_T = \\ D^\alpha I_T = D^\alpha I_i = D^\alpha I_N = D^\alpha R = D^\alpha D_u = D^\alpha S_r = D^\alpha E_r = D^\alpha I_r = 0 \end{array}\right\} \quad (3)$$

The disease free equilibrium (U^e):

$$\{S^e, S_v^e, S_u^e, S_{vc}^e, S_{vn}^e, S_{uc}^e, S_{un}^e, E_Q^e, E_T^e, I_T^e, I_i^e, I_N^e, R^e, D_u^e, S_r^e, E_r^e, I_r^e\} = \{\frac{P}{\mu+r+K_1X}, \frac{rP}{(\mu+r+K_1X)(\mu+\lambda+K_2Z)}, \frac{K_1XP}{(\mu+r+K_1X)(\mu+\sigma+K_3Y)}, \frac{\lambda rP}{(\mu+r+K_1X)(\mu+\lambda+K_2Z)(\mu+\beta_1)}, \frac{K_2ZrP}{(\mu+r+K_1X)(\mu+\lambda+K_2Z)(\mu+\beta_2)}, \frac{\sigma K_1XP}{(\mu+r+K_1X)(\mu+\sigma+K_3Y)(\mu+\beta_3)}, \frac{K_3YK_1XP}{(\mu+r+K_1X)(\mu+\sigma+K_3Y)(\mu+\beta_4)}, 0,0,0,0,0,0,0,0,0, \frac{\Lambda}{\mu_r}, 0,0\}.$$

The endemic equilibrium $(U^c) = \{S^c, S_v^c, S_u^c, S_{vc}^c, S_{vn}^c, S_{uc}^c, S_{un}^c, E_Q^c, E_T^c, I_T^c, I_i^c, I_N^c, R^c, D_u^c, S_r^c, E_r^c, I_r^c\} \neq \{0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0\}$

Where;

$$S^c = \frac{P+\tau R^c}{\mu+r+K_1X}, S_v^c = \frac{r(P+\tau R^c)}{(\mu+r+K_1X)(\mu+\lambda+K_2Z)}, S_u^c = \frac{K_1X(P+\tau R^c)}{(\mu+r+K_1X)(\mu+\sigma+K_3Y)}, S_{vc}^c = \frac{\lambda r(P+\tau R^c)}{(\mu+r+K_1X)(\mu+\lambda+K_2Z)(\mu+\beta_1)},$$

$$S_{vn}^c = \frac{K_2Zr(P+\tau R^c)}{(\mu+r+K_1X)(\mu+\lambda+K_2Z)(\mu+\beta_2)}, S_{uc}^c = \frac{\sigma K_1X(P+\tau R^c)}{(\mu+r+K_1X)(\mu+\sigma+K_3Y)(\mu+\beta_3)}, S_{un}^c = \frac{K_3YK_1X(P+\tau R^c)}{(\mu+r+K_1X)(\mu+\sigma+K_3Y)(\mu+\beta_4)},$$

$$E^c = \frac{P+\tau R^c}{(\mu+\alpha_1+\alpha_2)(\mu+r+K_1X)}\left[\frac{r[\beta_1\lambda(\mu+\beta_2)+\beta_2K_2Z(\mu+\beta_1)]}{(\mu+\beta_1)(\mu+\beta_2)(\mu+\lambda+K_2Z)} + \frac{K_1X[\beta_3\sigma(\mu+\beta_4)+\beta_4K_3Y(\mu+\beta_3)]}{(\mu+\beta_3)(\mu+\beta_4)(\mu+\sigma+K_3Y)}\right],$$

$$E_Q^c = \frac{\alpha_1(P+\tau R^c)}{(\mu+\theta_1)(\mu+\alpha_1+\alpha_2)(\mu+r+K_1X)}\left[\frac{r[\beta_1\lambda(\mu+\beta_2)+\beta_2K_2Z(\mu+\beta_1)]}{(\mu+\beta_1)(\mu+\beta_2)(\mu+\lambda+K_2Z)} + \frac{K_1X[\beta_3\sigma(\mu+\beta_4)+\beta_4K_3Y(\mu+\beta_3)]}{(\mu+\beta_3)(\mu+\beta_4)(\mu+\sigma+K_3Y)}\right],$$

$$E_T^c = \frac{\alpha_2(P+\tau R^c)}{(\mu+\theta_2+\rho)(\mu+\alpha_1+\alpha_2)(\mu+r+K_1X)}\left[\frac{r[\beta_1\lambda(\mu+\beta_2)+\beta_2K_2Z(\mu+\beta_1)]}{(\mu+\beta_1)(\mu+\beta_2)(\mu+\lambda+K_2Z)} + \frac{K_1X[\beta_3\sigma(\mu+\beta_4)+\beta_4K_3Y(\mu+\beta_3)]}{(\mu+\beta_3)(\mu+\beta_4)(\mu+\sigma+K_3Y)}\right],$$

$$I_T^c = \frac{(P+\tau R^c)}{(d_1+J_2)(\mu+\alpha_1+\alpha_2)(\mu+r+K_1X)}\left[\left[\frac{r[\beta_1\lambda(\mu+\beta_2)+\beta_2K_2Z(\mu+\beta_1)]}{(\mu+\beta_1)(\mu+\beta_2)(\mu+\lambda+K_2Z)} + \frac{K_1X[\beta_3\sigma(\mu+\beta_4)+\beta_4K_3Y(\mu+\beta_3)]}{(\mu+\beta_3)(\mu+\beta_4)(\mu+\sigma+K_3Y)}\right]\left[\frac{C_1\alpha_1}{(\mu+\theta_1)} + \frac{C_2\alpha_2}{(\mu+\theta_2+\rho)}\right]\right],$$

$$I_i^c = \frac{(P+\tau R^c)}{(d_2+J_1)(\mu+\alpha_1+\alpha_2)(\mu+r+K_1X)}\left[\left[\frac{r[\beta_1\lambda(\mu+\beta_2)+\beta_2K_2Z(\mu+\beta_1)]}{(\mu+\beta_1)(\mu+\beta_2)(\mu+\lambda+K_2Z)}+\frac{K_1X[\beta_3\sigma(\mu+\beta_4)+\beta_4K_3Y(\mu+\beta_3)]}{(\mu+\beta_3)(\mu+\beta_4)(\mu+\sigma+K_3Y)}\right]\left[\frac{C_3\alpha_1}{(\mu+\theta_1)}+\frac{C_4\alpha_2}{(\mu+\theta_2+\rho)}\right]\right],$$

$$I_N^c = \frac{(P+\tau R^c)}{d_3(\mu+\alpha_1+\alpha_2)(\mu+r+K_1X)}\left[\left[\frac{r[\beta_1\lambda(\mu+\beta_2)+\beta_2K_2Z(\mu+\beta_1)]}{(\mu+\beta_1)(\mu+\beta_2)(\mu+\lambda+K_2Z)}+\frac{K_1X[\beta_3\sigma(\mu+\beta_4)+\beta_4K_3Y(\mu+\beta_3)]}{(\mu+\beta_3)(\mu+\beta_4)(\mu+\sigma+K_3Y)}\right]\left[\frac{C_5\alpha_1}{(\mu+\theta_1)}+\frac{C_6\alpha_2}{(\mu+\theta_2+\rho)}\right]\right],$$

$$R^c = \frac{PW}{\tau(1-W)}, \text{ where } W = \left[\left[\frac{J_1}{(d_2+J_1)(\mu+\alpha_1+\alpha_2)(\mu+r+K_1X)}\left[\frac{C_3\alpha_1}{(\mu+\theta_1)}+\frac{C_4\alpha_2}{(\mu+\theta_2+\rho)}\right]+\frac{J_2}{(d_1+J_2)(\mu+\alpha_1+\alpha_2)(\mu+r+K_1X)}\left[\frac{C_1\alpha_1}{(\mu+\theta_1)}+\frac{C_2\alpha_2}{(\mu+\theta_2+\rho)}\right]+\frac{\rho\alpha_2}{(\mu+\theta_2+\rho)(\mu+\alpha_1+\alpha_2)(\mu+r+K_1X)}\right]\left[\frac{r[\beta_1\lambda(\mu+\beta_2)+\beta_2K_2Z(\mu+\beta_1)]}{(\mu+\beta_1)(\mu+\beta_2)(\mu+\lambda+K_2Z)}+\frac{K_1X[\beta_3\sigma(\mu+\beta_4)+\beta_4K_3Y(\mu+\beta_3)]}{(\mu+\beta_3)(\mu+\beta_4)(\mu+\sigma+K_3Y)}\right]\right],$$

$$S_r^c = \frac{\Lambda}{\mu_r},\ E_r^c = \frac{\omega\Lambda}{\mu_r^2},\ I_r^c = \frac{\Phi\omega\Lambda}{d_4\mu_r^2}.$$

With effective reproduction number (R^Q) represented as follows; [21, 22, 23].

$$R^Q = \frac{L(bef+cdg)}{aceh}+\frac{K(bel+cdm)}{acen}-\frac{H(befknt+behinu+cdgknt+behklv+cdhjnu+cdhkmv)}{acehknw}$$

$$\text{where: } L = \frac{m_2S_{vc}+n_2S_{vn}+e_2S_{uc}+t_2S_{un}}{N}, K = \frac{m_1S_{vc}+n_1S_{vn}+e_1S_{uc}+t_1S_{un}}{N},\ H = \frac{m_4S_{vc}+n_4S_{vn}+e_4S_{uc}+t_4S_{un}}{N},\ a = (\mu+\alpha_1+\alpha_2), b = -\alpha_2, c = (\mu+\theta_1), d = -\alpha_1, e = (\mu+\theta_2+\rho), f = -C_1, g = -C_2, h = (d_1+J_2), k = (d_2+J_1), i = -C_3, j = -C_4, l = -C_5, m = -C_6, p = -J_1, q = -J_2, n = d_3, r = -\rho, s = \tau, t = -d_1, u = -d_2, v = -d_3, z = d_4, w = q, x = \mu_r, y = \phi.$$

Thus, U^e is locally asymptotically stable when $R^Q < 1$ and unstable if otherwise. While, U^c is locally asymptotically stable when $R^Q > 1$ and unstable if otherwise.

Theorem 1

There is a unique solution for the initial value problem given in system (1) and the solution remains in R^{18}, $x \geq 0$.

Proof

We need to show that the domain R^{18}, $x \geq 0$ is positively invariant.

Since,

$$\left.\begin{array}{c}
D^\alpha S(t)|_{S=0} = P+\tau R \geq 0\\
D^\alpha S_v(t)|_{S_v=0} = rS \geq 0\\
D^\alpha S_u(t)|_{S_u=0} = K_1XS \geq 0\\
D^\alpha S_{vc}(t)|_{S_{vc}=0} = \lambda S_v \geq 0\\
D^\alpha S_{vn}(t)|_{S_{vn}=0} = K_2ZS_v \geq 0\\
D^\alpha S_{uc}(t)|_{S_{uc}=0} = \sigma S_u \geq 0\\
D^\alpha S_{un}(t)|_{S_{un}=0} = K_3YS_u \geq 0\\
D^\alpha E(t)|_{E=0} = \frac{1}{N}[m_1I_NS_{vc}+m_2I_TS_{vc}+m_3I_rS_{vc}+m_4D_uS_{vc}+\\
n_1I_NS_{vn}+n_2I_TS_{vn}+n_3I_rS_{vn}+n_4D_uS_{vn}+e_1I_NS_{uc}+e_2I_TS_{uc}+e_3I_rS_{uc}+e_4D_uS_{uc}+\\
t_1I_NS_{un}+t_2I_TS_{un}+t_3I_rS_{un}+t_4D_uS_{un}] \geq 0\\
D^\alpha E_Q(t)|_{E_Q=0} = \alpha_1E \geq 0\\
D^\alpha E_T(t)|_{E_T=0}\ \alpha_2E \geq 0\\
D^\alpha I_T(t)|_{I_T=0} = C_1E_Q+C_2E_T \geq 0\\
D^\alpha I_i(t)|_{I_i=0} = C_3E_Q+C_4E_T \geq 0\\
D^\alpha I_N(t)|_{I_N=0} = C_5E_Q+C_6E_T \geq 0\\
D^\alpha R(t)|_{R=0} = J_1I_i+J_2I_T+\rho E_T \geq 0\\
D^\alpha D_u(t)|_{D_u=0} = d_3I_N+d_2I_i+d_1I_T \geq 0\\
D^\alpha S_r(t)|_{S_r=0} = \Lambda \geq 0\\
D^\alpha E_r(t)|_{E_r=0} = \omega S_r \geq 0\\
D^\alpha I_r(t)|_{I_r=0} = \phi E_r \geq 0
\end{array}\right\} \quad (4)$$

We then conclude that the solution of the system (1) lies in the feasible domain.

Therefore, the uniqueness and solution of the system (1) exists.

Approximate Solution of the Model Using LADM

In this section, we apply the Laplace Adomian Decomposition Method (LADM) technique to the fractional order EVD model (1).

Applying Laplace transform to both sides of (1) gives;

$$\left.\begin{aligned}
S^{\alpha}\mathcal{L}\{S\} - S^{\alpha-1}S(0) &= \mathcal{L}\{P - (\mu + r + K_1X)S + \tau R\} \\
S^{\alpha}\mathcal{L}\{S_v\} - S^{\alpha-1}S_v(0) &= \mathcal{L}\{rS - (\mu + \lambda + K_2Z)S_v\} \\
S^{\alpha}\mathcal{L}\{S_u\} - S^{\alpha-1}S_u(0) &= \mathcal{L}\{K_1XS - (\mu + \sigma + K_3Y)S_u\} \\
S^{\alpha}\mathcal{L}\{S_{vc}\} - S^{\alpha-1}S_{vc}(0) &= \mathcal{L}\left\{\lambda S_v - \left[\mu S_{vc} + \frac{1}{N}(m_1I_NS_{vc} + m_2I_TS_{vc} + m_3I_rS_{vc} + m_4D_uS_{vc})\right]\right\} \\
S^{\alpha}\mathcal{L}\{S_{vn}\} - S^{\alpha-1}S_{vn}(0) &= \mathcal{L}\left\{K_2ZS_v - \left[\mu S_{vn} + \frac{1}{N}(n_1I_NS_{vn} + n_2I_TS_{vn} + n_3I_rS_{vn} + n_4D_uS_{vn})\right]\right\} \\
S^{\alpha}\mathcal{L}\{S_{uc}\} - S^{\alpha-1}S_{uc}(0) &= \mathcal{L}\left\{\sigma S_u - \left[S_{uc}\mu + \frac{1}{N}(e_1I_NS_{uc} + e_2I_TS_{uc} + e_3I_rS_{uc} + e_4D_uS_{uc})\right]\right\} \\
S^{\alpha}\mathcal{L}\{S_{un}\} - S^{\alpha-1}S_{un}(0) &= \mathcal{L}\left\{K_3YS_u - \left[\mu S_{un} + \frac{1}{N}(t_1I_NS_{un} + t_2I_TS_{un} + t_3I_rS_{un} + t_4D_uS_{un})\right]\right\} \\
S^{\alpha}\mathcal{L}\{E\} - S^{\alpha-1}E(0) &= \mathcal{L}\left\{\frac{1}{N}[m_1I_NS_{vc} + m_2I_TS_{vc} + m_3I_rS_{vc} + m_4D_uS_{vc} + n_1I_NS_{vn} + \right. \\
&\quad n_2I_TS_{vn} + n_3I_rS_{vn} + n_4D_uS_{vn} + e_1I_NS_{uc} + e_2I_TS_{uc} + e_3I_rS_{uc} + e_4D_uS_{uc} + t_1I_NS_{un} + \\
&\quad \left. t_2I_TS_{un} + t_3I_rS_{un} + t_4D_uS_{un}] - (\mu + \alpha_1 + \alpha_2)E\right\} \\
S^{\alpha}\mathcal{L}\{E_Q\} - S^{\alpha-1}E_Q(0) &= \mathcal{L}\{\alpha_1E - (\mu + \theta_1)E_Q\} \\
S^{\alpha}\mathcal{L}\{E_T\} - S^{\alpha-1}E_T(0) &= \mathcal{L}\{\alpha_2E - (\mu + \theta_2 + \rho)E_T\} \\
S^{\alpha}\mathcal{L}\{I_T\} - S^{\alpha-1}I_T(0) &= \mathcal{L}\{C_1E_Q + C_2E_T - (d_1 + J_2)I_T\} \\
S^{\alpha}\mathcal{L}\{I_i\} - S^{\alpha-1}I_i(0) &= \mathcal{L}\{C_2E_Q + C_4E_T - (d_2 + J_1)I_i\} \\
S^{\alpha}\mathcal{L}\{I_N\} - S^{\alpha-1}I_N(0) &= \mathcal{L}\{C_5E_Q + C_6E_T - d_3I_N\} \\
S^{\alpha}\mathcal{L}\{R\} - S^{\alpha-1}R(0) &= \mathcal{L}\{J_1I_i + J_2I_T + \rho E_T - \tau R\} \\
S^{\alpha}\mathcal{L}\{D_u\} - S^{\alpha-1}D_u(0) &= \mathcal{L}\{d_3I_N + d_2I_i + d_1I_T\} \\
S^{\alpha}\mathcal{L}\{S_r\} - S^{\alpha-1}S_r(0) &= \mathcal{L}\{\Lambda - \mu_rS_r\} \\
S^{\alpha}\mathcal{L}\{E_r\} - S^{\alpha-1}E_r(0) &= \mathcal{L}\{\omega S_r - \mu_rE_r\} \\
S^{\alpha}\mathcal{L}\{I_r\} - S^{\alpha-1}I_r(0) &= \mathcal{L}\{\phi E_r - d_4I_r\}
\end{aligned}\right\} \quad (5)$$

Applying the initial conditions (2) to (5) and dividing by S^{α} gives the following;

$$\left.\begin{aligned}
\mathcal{L}\{S\} &= \frac{g_1}{S} + \frac{1}{S^{\alpha}}[\mathcal{L}\{P - (\mu + r + K_1X)S + \tau R\}] \\
\mathcal{L}\{S_v\} &= \frac{g_2}{S} + \frac{1}{S^{\alpha}}[\mathcal{L}\{rS - (\mu + \lambda + K_2Z)S_v\}] \\
\mathcal{L}\{S_u\} &= \frac{g_3}{S} + \frac{1}{S^{\alpha}}[\mathcal{L}\{K_1XS - (\mu + \sigma + K_3Y)S_u\} \\
\mathcal{L}\{S_{vc}\} &\frac{g_4}{S} + \frac{1}{S^{\alpha}}[\mathcal{L}\{\lambda S_v - [\mu S_{vc} + \frac{1}{N}(m_1I_NS_{vc} + m_2I_TS_{vc} + m_3I_rS_{vc} + m_4D_uS_{vc})]\}] \\
\mathcal{L}\{S_{vn}\} &= \frac{g_5}{S} + \frac{1}{S^{\alpha}}[\mathcal{L}\{K_2ZS_v - [\mu S_{vn} + \frac{1}{N}(n_1I_NS_{vn} + n_2I_TS_{vn} + n_3I_rS_{vn} + n_4D_uS_{vn})]\}] \\
\mathcal{L}\{S_{uc}\} &= \frac{g_6}{S} + \frac{1}{S^{\alpha}}[\mathcal{L}\{\sigma S_u - [\mu S_{uc} + \frac{1}{N}(e_1I_NS_{uc} + e_2I_TS_{uc} + e_3I_rS_{uc} + e_4D_uS_{uc})]\}] \\
\mathcal{L}\{S_{un}\} &= \frac{g_7}{S} + \frac{1}{S^{\alpha}}[\mathcal{L}\{K_3YS_u - [\mu S_{un} + \frac{1}{N}(t_1I_NS_{un} + t_2I_TS_{un} + t_3I_rS_{un} + t_4D_uS_{un})]\}] \\
\mathcal{L}\{E\} &= \frac{g_8}{S} + \frac{1}{S^{\alpha}}[\mathcal{L}\{\frac{1}{N}[m_1I_NS_{vc} + m_2I_TS_{vc} + m_3I_rS_{vc} + m_4D_uS_{vc} + \\
&\quad n_1I_NS_{vn} + n_2I_TS_{vn} + n_3I_rS_{vn} + n_4D_uS_{vn} + e_1I_NS_{uc} + e_2I_TS_{uc} + e_3I_rS_{uc} + e_4D_uS_{uc} + \\
&\quad t_1I_NS_{un} + t_2I_TS_{un} + t_3I_rS_{un} + t_4D_uS_{un}] - (\mu + \alpha_1 + \alpha_2)E\}] \\
\mathcal{L}\{E_Q\} &= \frac{g_9}{S} + \frac{1}{S^{\alpha}}[\mathcal{L}\{\alpha_1E - (\mu + \theta_1)E_Q\}] \\
\mathcal{L}\{E_T\} &= \frac{g_{10}}{S} + \frac{1}{S^{\alpha}}[\mathcal{L}\{\alpha_2E - (\mu + \theta_2 + \rho)E_T\}] \\
\mathcal{L}\{I_T\} &= \frac{g_{11}}{S} + \frac{1}{S^{\alpha}}[\mathcal{L}\{C_1E_Q + C_2E_T - (d_1 + J_2)I_T\}] \\
\mathcal{L}\{I_i\} &= \frac{g_{12}}{S} + \frac{1}{S^{\alpha}}[\mathcal{L}\{C_2E_Q + C_4E_T - (d_2 + J_1)I_i\}] \\
\mathcal{L}\{I_N\} &= \frac{g_{13}}{S} + \frac{1}{S^{\alpha}}[\mathcal{L}\{C_5E_Q + C_6E_T - d_3I_N\}] \\
\mathcal{L}\{R\} &= \frac{g_{14}}{S} + \frac{1}{S^{\alpha}}[\mathcal{L}\{J_1I_i + J_2I_T + \rho E_T - \tau R\}] \\
\mathcal{L}\{D_u\} &= \frac{g_{15}}{S} + \frac{1}{S^{\alpha}}[\mathcal{L}\{d_3I_N + d_2I_i + d_1I_T\}] \\
\mathcal{L}\{S_r\} &= \frac{g_{16}}{S} + \frac{1}{S^{\alpha}}[\mathcal{L}\{\Lambda - \mu_rS_r\}] \\
\mathcal{L}\{E_r\} &= \frac{g_{17}}{S} + \frac{1}{S^{\alpha}}[\mathcal{L}\{\omega S_r - \mu_rE_r\}] \\
\mathcal{L}\{I_r\} &= \frac{g_{18}}{S} + \frac{1}{S^{\alpha}}[\mathcal{L}\{\phi E_r - d_4I_r\}]
\end{aligned}\right\} \quad (6)$$

Assuming that the solutions of $S(t)$, $S_v(t)$, $S_u(t)$, $S_{vc}(t)$, $S_{vn}(t)$, $S_{uc}(t)$, $S_{un}(t)$, $E(t)$, $E_Q(t)$, $E_T(t)$, $I_T(t)$, $I_i(t)$, $I_N(t)$, $R(t)$, $D_u(t)$, $S_r(t)$, $E_r(t)$, $I_r(t)$ are

in the form of infinite series given by;

$$\left.\begin{aligned}
S(t) &= \sum_{n=0}^{\infty} S_n \\
S_v(t) &= \sum_{n=0}^{\infty} S_{v_n} \\
S_u(t) &= \sum_{n=0}^{\infty} S_{u_n} \\
S_{vc}(t) &= \sum_{n=0}^{\infty} S_{vc_n} \\
S_{vn}(t) &= \sum_{n=0}^{\infty} S_{vn_n} \\
S_{uc}(t) &= \sum_{n=0}^{\infty} S_{uc_n} \\
S_{un}(t) &= \sum_{n=0}^{\infty} S_{un_n} \\
E(t) &= \sum_{n=0}^{\infty} E_n \\
E_Q(t) &= \sum_{n=0}^{\infty} E_{Q_n} \\
E_T(t) &= \sum_{n=0}^{\infty} E_{T_n} \\
I_T(t) &= \sum_{n=0}^{\infty} I_{T_n} \\
I_l(t) &= \sum_{n=0}^{\infty} I_{l_n} \\
I_N(t) &= \sum_{n=0}^{\infty} I_{N_n} \\
R(t) &= \sum_{n=0}^{\infty} R_n \\
D_u(t) &= \sum_{n=0}^{\infty} D_{u_n} \\
S_r(t) &= \sum_{n=0}^{\infty} S_{r_n} \\
E_r(t) &= \sum_{n=0}^{\infty} E_{r_n} \\
I_r(t) &= \sum_{n=0}^{\infty} I_{r_n}
\end{aligned}\right\} \quad (7)$$

We then decompose the nonlinear terms of the model before we proceed as follows;

$$\left.\begin{aligned}
I_N(t)S_{vc}(t) &= \sum_{n=0}^{\infty} A_n \\
I_T(t)S_{vc}(t) &= \sum_{n=0}^{\infty} B_n \\
I_r(t)S_{vc}(t) &= \sum_{n=0}^{\infty} F_n \\
D_u(t)S_{vc}(t) &= \sum_{n=0}^{\infty} G_n \\
I_N(t)S_{vn}(t) &= \sum_{n=0}^{\infty} H_n \\
I_T(t)S_{vn}(t) &= \sum_{n=0}^{\infty} L_n \\
I_r(t)S_{vn}(t) &= \sum_{n=0}^{\infty} Q_n \\
D_u(t)S_{vn}(t) &= \sum_{n=0}^{\infty} T_n \\
I_N(t)S_{uc}(t) &= \sum_{n=0}^{\infty} U_n \\
I_T(t)S_{uc}(t) &= \sum_{n=0}^{\infty} V_n \\
I_r(t)S_{uc}(t) &= \sum_{n=0}^{\infty} W_n \\
D_u(t)S_{uc}(t) &= \sum_{n=0}^{\infty} X_n \\
I_N(t)S_{un}(t) &= \sum_{n=0}^{\infty} Y_n \\
I_T(t)S_{un}(t) &= \sum_{n=0}^{\infty} Z_n \\
I_r(t)S_{un}(t) &= \sum_{n=0}^{\infty} N_n \\
D_u(t)S_{un}(t) &= \sum_{n=0}^{\infty} M_n
\end{aligned}\right\} \quad (8)$$

Where the above Adomian polynomials are defined as follows;

$$\left.\begin{aligned}
A_n &= \frac{1}{\Gamma(n+1)}\frac{d^n}{d\lambda^n}\{\sum_{k=0}^{n}\lambda^k I_{Nk}(t)\sum_{k=0}^{n}\lambda^k S_{vck}(t)\}|_{\lambda=0} \\
B_n &= \frac{1}{\Gamma(n+1)}\frac{d^n}{d\lambda^n}\{\sum_{k=0}^{n}\lambda^k I_{Tk}(t)\sum_{k=0}^{n}\lambda^k S_{vck}(t)\}|_{\lambda=0} \\
F_n &= \frac{1}{\Gamma(n+1)}\frac{d^n}{d\lambda^n}\{\sum_{k=0}^{n}\lambda^k I_{rk}(t)\sum_{k=0}^{n}\lambda^k S_{vck}(t)\}|_{\lambda=0} \\
G_n &= \frac{1}{\Gamma(n+1)}\frac{d^n}{d\lambda^n}\{\sum_{k=0}^{n}\lambda^k D_{uk}(t)\sum_{k=0}^{n}\lambda^k S_{vck}(t)\}|_{\lambda=0} \\
H_n &= \frac{1}{\Gamma(n+1)}\frac{d^n}{d\lambda^n}\{\sum_{k=0}^{n}\lambda^k I_{Nk}(t)\sum_{k=0}^{n}\lambda^k S_{vnk}(t)\}|_{\lambda=0} \\
L_n &= \frac{1}{\Gamma(n+1)}\frac{d^n}{d\lambda^n}\{\sum_{k=0}^{n}\lambda^k I_{Tk}(t)\sum_{k=0}^{n}\lambda^k S_{vnk}(t)\}|_{\lambda=0} \\
Q_n &= \frac{1}{\Gamma(n+1)}\frac{d^n}{d\lambda^n}\{\sum_{k=0}^{n}\lambda^k I_{rk}(t)\sum_{k=0}^{n}\lambda^k S_{vnk}(t)\}|_{\lambda=0} \\
T_n &= \frac{1}{\Gamma(n+1)}\frac{d^n}{d\lambda^n}\{\sum_{k=0}^{n}\lambda^k D_{uk}(t)\sum_{k=0}^{n}\lambda^k S_{vnk}(t)\}|_{\lambda=0} \\
U_n &= \frac{1}{\Gamma(n+1)}\frac{d^n}{d\lambda^n}\{\sum_{k=0}^{n}\lambda^k I_{Nk}(t)\sum_{k=0}^{n}\lambda^k S_{uck}(t)\}|_{\lambda=0} \\
V_n &= \frac{1}{\Gamma(n+1)}\frac{d^n}{d\lambda^n}\{\sum_{k=0}^{n}\lambda^k I_{Tk}(t)\sum_{k=0}^{n}\lambda^k S_{uck}(t)\}|_{\lambda=0} \\
W_n &= \frac{1}{\Gamma(n+1)}\frac{d^n}{d\lambda^n}\{\sum_{k=0}^{n}\lambda^k I_{rk}(t)\sum_{k=0}^{n}\lambda^k S_{uck}(t)\}|_{\lambda=0} \\
X_n &= \frac{1}{\Gamma(n+1)}\frac{d^n}{d\lambda^n}\{\sum_{k=0}^{n}\lambda^k D_{uk}(t)\sum_{k=0}^{n}\lambda^k S_{uck}(t)\}|_{\lambda=0} \\
Y_n &= \frac{1}{\Gamma(n+1)}\frac{d^n}{d\lambda^n}\{\sum_{k=0}^{n}\lambda^k I_{Nk}(t)\sum_{k=0}^{n}\lambda^k S_{unk}(t)\}|_{\lambda=0} \\
Z_n &= \frac{1}{\Gamma(n+1)}\frac{d^n}{d\lambda^n}\{\sum_{k=0}^{n}\lambda^k I_{Tk}(t)\sum_{k=0}^{n}\lambda^k S_{unk}(t)\}|_{\lambda=0} \\
N_n &= \frac{1}{\Gamma(n+1)}\frac{d^n}{d\lambda^n}\{\sum_{k=0}^{n}\lambda^k I_{rk}(t)\sum_{k=0}^{n}\lambda^k S_{unk}(t)\}|_{\lambda=0} \\
M_n &= \frac{1}{\Gamma(n+1)}\frac{d^n}{d\lambda^n}\{\sum_{k=0}^{n}\lambda^k D_{uk}(t)\sum_{k=0}^{n}\lambda^k S_{unk}(t)\}|_{\lambda=0}
\end{aligned}\right\} \quad (9)$$

From the above definitions when $n = 0$ we have the following;

$$\left.\begin{aligned}
A_0 &= \frac{1}{\Gamma(1)}\frac{d^0}{d\lambda^0}\{[\lambda^0 I_{N0}][\lambda^0 S_{vc}\ ]\} = I_{N0}S_{vc0}\\
B_0 &= \frac{1}{\Gamma(1)}\frac{d^0}{d\lambda^0}\{[\lambda^0 I_{T0}][\lambda^0 S_{vc0}]\} = I_{T0}S_{vc0}\\
F_0 &= \frac{1}{\Gamma(1)}\frac{d^0}{d\lambda^0}\{[\lambda^0 I_{r0}][\lambda^0 S_{vc0}]\} = I_{r0}S_{vc0}\\
G_0 &= \frac{1}{\Gamma(1)}\frac{d^0}{d\lambda^0}\{[\lambda^0 D_{u0}][\lambda^0 S_{vc0}]\} = D_{u0}S_{vc0}\\
H_0 &= \frac{1}{\Gamma(1)}\frac{d^0}{d\lambda^0}\{[\lambda^0 I_{N0}][\lambda^0 S_{vn0}]\} = I_{N0}S_{vn0}\\
L_0 &= \frac{1}{\Gamma(1)}\frac{d^0}{d\lambda^0}\{[\lambda^0 I_{T0}][\lambda^0 S_{vn0}]\} = I_{T0}S_{vn0}\\
Q_0 &= \frac{1}{\Gamma(1)}\frac{d^0}{d\lambda^0}\{[\lambda^0 I_{r0}][\lambda^0 S_{vn0}]\} = I_{r0}S_{vn0}\\
T_0 &= \frac{1}{\Gamma(1)}\frac{d^0}{d\lambda^0}\{[\lambda^0 D_{u0}][\lambda^0 S_{vn0}]\} = D_{u0}S_{vn0}\\
U_0 &= \frac{1}{\Gamma(1)}\frac{d^0}{d\lambda^0}\{[\lambda^0 I_{N0}][\lambda^0 S_{uc0}]\} = I_{N0}S_{uc0}\\
V_0 &= \frac{1}{\Gamma(1)}\frac{d^0}{d\lambda^0}\{[\lambda^0 I_{T0}][\lambda^0 S_{uc0}]\} = I_{T0}S_{uc0}\\
W_0 &= \frac{1}{\Gamma(1)}\frac{d^0}{d\lambda^0}\{[\lambda^0 I_{r0}][\lambda^0 S_{uc0}]\} = I_{r0}S_{uc0}\\
X_0 &= \frac{1}{\Gamma(1)}\frac{d^0}{d\lambda^0}\{[\lambda^0 D_{u0}][\lambda^0 S_{uc0}]\} = D_{u0}S_{uc}\\
Y_0 &= \frac{1}{\Gamma(1)}\frac{d^0}{d\lambda^0}\{[\lambda^0 I_{N0}][\lambda^0 S_{un0}]\} = I_{N0}S_{un0}\\
Z_0 &= \frac{1}{\Gamma(1)}\frac{d^0}{d\lambda^0}\{[\lambda^0 I_{T0}][\lambda^0 S_{un0}]\} = I_{T0}S_{un0}\\
N_0 &= \frac{1}{\Gamma(1)}\frac{d^0}{d\lambda^0}\{[\lambda^0 I_{r0}][\lambda^0 S_{un0}]\} = I_{r0}S_{un0}\\
M_0 &= \frac{1}{\Gamma(1)}\frac{d^0}{d\lambda^0}\{[\lambda^0 D_{u0}][\lambda^0 S_{un}\ ]\} = D_{u0}S_{un0}
\end{aligned}\right\} \quad (10)$$

Similarly, when n =1 we have the following;

$$\left.\begin{aligned}
A_1 &= \frac{1}{\Gamma(2)}\frac{d}{d\lambda}\{[I_{N0} + \lambda I_{N1}][S_{vc0} + \lambda S_{vc1}]\}|_{\lambda=0} = I_{N0}S_{vc1} + I_{N1}S_{vc0}\\
B_1 &= \frac{1}{\Gamma(2)}\frac{d}{d\lambda}\{[I_{T0} + \lambda I_{T1}][S_{vc0} + \lambda S_{vc1}]\}|_{\lambda=0} = I_{T0}S_{vc1} + I_{T1}S_{vc0}\\
F_1 &= \frac{1}{\Gamma(2)}\frac{d}{d\lambda}\{[I_{r0} + \lambda I_{r1}][S_{vc0} + \lambda S_{vc1}]\}|_{\lambda=0} = I_{r0}S_{vc}\ + I_{r1}S_{vc0}\\
G_1 &= \frac{1}{\Gamma(2)}\frac{d}{d\lambda}\{[D_{u0} + \lambda D_{u1}][S_{vc0} + \lambda S_{vc1}]\}|_{\lambda=0} = D_{u0}S_{vc1} + D_{u1}S_{vc0}\\
H_1 &= \frac{1}{\Gamma(2)}\frac{d}{d\lambda}\{[I_{N0} + \lambda I_{N1}][S_{vn0} + \lambda S_{vn1}]\}|_{\lambda=0} = I_{N0}S_{vn}\ + I_{N1}S_{vn0}\\
L_1 &= \frac{1}{\Gamma(2)}\frac{d}{d\lambda}\{[I_{T0} + \lambda I_{T1}][S_{vn0} + \lambda S_{vn1}]\}|_{\lambda=0} = I_{T0}S_{vn1} + I_{T1}S_{vn0}\\
Q_1 &= \frac{1}{\Gamma(2)}\frac{d}{d\lambda}\{[I_{r0} + \lambda I_{r1}][S_{vn0} + \lambda S_{vn1}]\}|_{\lambda=0} = I_{r0}S_{vn1} + I_{r1}S_{vn0}\\
T_1 &= \frac{1}{\Gamma(2)}\frac{d}{d\lambda}\{[D_{u0} + \lambda D_{u1}][S_{vn}\ + \lambda S_{vn1}]\}|_{\lambda=0} = D_{u0}S_{vn1} + D_{u1}S_{vn}\\
U_1 &= \frac{1}{\Gamma(2)}\frac{d}{d\lambda}\{[I_{N0} + \lambda I_{N1}][S_{uc0} + \lambda S_{uc1}]\}|_{\lambda=0} = I_{N0}S_{uc1} + I_{N1}S_{uc0}\\
V_1 &= \frac{1}{\Gamma(2)}\frac{d}{d\lambda}\{[I_{T0} + \lambda I_{T1}][S_{uc}\ + \lambda S_{uc1}]\}|_{\lambda=0} = I_{T0}S_{uc}\ + I_{T1}S_{uc0}\\
W_1 &= \frac{1}{\Gamma(2)}\frac{d}{d\lambda}\{[I_{r0} + \lambda I_{r1}][S_{uc}\ + \lambda S_{uc1}]\}|_{\lambda=0} = I_{r0}S_{uc}\ + I_{r1}S_{uc0}\\
X_1 &= \frac{1}{\Gamma(2)}\frac{d}{d\lambda}\{[D_{u0} + \lambda D_{u1}][S_{uc}\ + \lambda S_{uc}\ ]\}|_{\lambda=0} = D_{u0}S_{uc}\ + D_{u1}S_{uc}\\
Y_1 &= \frac{1}{\Gamma(2)}\frac{d}{d\lambda}\{[I_{N0} + \lambda I_{N1}][S_{un}\ + \lambda S_{un1}]\}|_{\lambda=0} = I_{N0}S_{un}\ + I_{N1}S_{un0}\\
Z_1 &= \frac{1}{\Gamma(2)}\frac{d}{d\lambda}\{[I_{T0} + \lambda I_{T1}][S_{un0} + \lambda S_{un}\ ]\}|_{\lambda=0} = I_{T0}S_{un}\ + I_{T1}S_{un}\\
N_1 &= \frac{1}{\Gamma(2)}\frac{d}{d\lambda}\{[I_{r0} + \lambda I_{r1}][S_{un}\ + \lambda S_{un}\ ]\}|_{\lambda=0} = I_{r0}S_{un}\ + I_{r1}S_{un0}\\
M_1 &= \frac{1}{\Gamma(2)}\frac{d}{d\lambda}\{[D_{u0} + \lambda D_{u1}][S_{un}\ + \lambda S_{un1}]\}|_{\lambda=0} = D_{u0}S_{un1} + D_{u1}S_{un}
\end{aligned}\right\} \quad (11)$$

Considering the above equations we have the following;

$$\left.\begin{aligned}
&\mathcal{L}\{S_0\} = \frac{g_1}{s}, \mathcal{L}\{S_{v0}\} = \frac{g_2}{s}, \mathcal{L}\{S_{u0}\} = \frac{g_3}{s}, \mathcal{L}\{S_{vc0}\} = \frac{g_4}{s}, \mathcal{L}\{S_{vn0}\} = \frac{g_5}{s},\\
&\mathcal{L}\{S_{uc0}\} = \frac{g_6}{s}, \mathcal{L}\{S_{un0}\} = \frac{g_7}{s}, \mathcal{L}\{E_0\} = \frac{g_8}{s}, \mathcal{L}\{E_{Q0}\} = \frac{g_9}{s}, \mathcal{L}\{E_{T0}\} = \frac{g_{10}}{s},\\
&\mathcal{L}\{I_{T0}\} = \frac{g_{11}}{s}, \mathcal{L}\{I_{i0}\} = \frac{g_{12}}{s}, \mathcal{L}\{I_{N0}\} = \frac{g_{13}}{s}, \mathcal{L}\{R_0\} = \frac{g_{14}}{s}, \mathcal{L}\{D_{u0}\} = \frac{g_{15}}{s},\\
&\mathcal{L}\{S_{r0}\} = \frac{g_{16}}{s}, \mathcal{L}\{E_{r0}\} = \frac{g_{17}}{s}, \mathcal{L}\{I_{r0}\} = \frac{g_{18}}{s}
\end{aligned}\right\} \quad (12)$$

$$
\left.
\begin{aligned}
\mathcal{L}\{S_1\} &= \frac{1}{s^\alpha}[\mathcal{L}\{P - (\mu + r + K_1X)S_0 + \tau R_0\}] \\
\mathcal{L}\{S_{v1}\} &= \frac{1}{s^\alpha}[\mathcal{L}\{rS_0 - (\mu + \lambda + K_2Z)S_{v0}\}] \\
\mathcal{L}\{S_{u1}\} &= \frac{1}{s^\alpha}[\mathcal{L}\{K_1XS_0 - (\mu + \sigma + K_3Y)S_{u0}\} \\
\mathcal{L}\{S_{vc1}\} &= \frac{1}{s^\alpha}[\mathcal{L}\left\{\lambda S_{v0} - [\mu S_{vc0} + \frac{1}{N}(m_1A_0 + m_2B_0 + m_3F_0 + m_4G_0)]\right\}] \\
\mathcal{L}\{S_{vn1}\} &= \frac{1}{s^\alpha}[\mathcal{L}\left\{K_2ZS_{v0} - [\mu S_{vn0} + \frac{1}{N}(n_1H_0 + n_2L_0 + n_3Q_0 + n_4T_0)]\right\}] \\
\mathcal{L}\{S_{uc}\} &= \frac{1}{s^\alpha}[\mathcal{L}\left\{\sigma S_{u0} - [\mu S_{uc0} + \frac{1}{N}(e_1U_0 + e_2V_0 + e_3W_0 + e_4X_0)]\right\}] \\
\mathcal{L}\{S_{un1}\} &= \frac{1}{s^\alpha}[\mathcal{L}\left\{K_3YS_{u0} - [\mu S_{un0} + \frac{1}{N}(t_1Y_0 + t_2Z_0 + t_3N_0 + t_4M_0)]\right\}] \\
\mathcal{L}\{E_1\} &= \frac{1}{s^\alpha}[\mathcal{L}\{\frac{1}{N}[m_1A_0 + m_2B_0 + m_3F_0 + m_4G_0 + \\
&\quad n_1H_0 + n_2L_0 + n_3Q_0 + n_4T_0 + e_1U_0 + e_2V_0 + e_3W_0 + e_4X_0 + \\
&\quad t_1Y_0 + t_2Z_0 + t_3N_0 + t_4M_0] - (\mu + \alpha_1 + \alpha_2)E_0\}] \\
\mathcal{L}\{E_{Q1}\} &= \frac{1}{s^\alpha}[\mathcal{L}\{\alpha_1E_0 - (\mu + \theta_1)E_{Q0}\}] \\
\mathcal{L}\{E_{T1}\} &= \frac{1}{s^\alpha}[\mathcal{L}\{\alpha_2E_0 - (\mu + \theta_2 + \rho)E_{T0}\}] \\
\mathcal{L}\{I_{T1}\} &= \frac{1}{s^\alpha}[\mathcal{L}\{C_1E_{Q0} + C_2E_{T0} - (d_1 + J_2)I_{T0}\}] \\
\mathcal{L}\{I_{i1}\} &= \frac{1}{s^\alpha}[\mathcal{L}\{C_2E_{Q0} + C_4E_{T0} - (d_2 + J_1)I_{i0}\}] \\
\mathcal{L}\{I_{N1}\} &= \frac{1}{s^\alpha}[\mathcal{L}\{C_5E_{Q0} + C_6E_{T0} - d_3I_{N0}\}] \\
\mathcal{L}\{R_1\} &= \frac{1}{s^\alpha}[\mathcal{L}\{J_1I_{i0} + J_2I_{T0} + \rho E_{T0} - \tau R_0\}] \\
\mathcal{L}\{D_{u1}\} &= \frac{1}{s^\alpha}[\mathcal{L}\{d_3I_{N0} + d_2I_{i0} + d_1I_{T0}\}] \\
\mathcal{L}\{S_{r1}\} &= \frac{1}{s^\alpha}[\mathcal{L}\{\Lambda - \mu_rS_{r0}\}] \\
\mathcal{L}\{E_{r1}\} &= \frac{1}{s^\alpha}[\mathcal{L}\{\omega S_{r0} - \mu_rE_{r0}\}] \\
\mathcal{L}\{I_{r1}\} &= \frac{1}{s^\alpha}[\mathcal{L}\{\phi E_{r0} - d_4I_{r0}\}]
\end{aligned}
\right\} \quad (13)
$$

$$
\left.
\begin{aligned}
\mathcal{L}\{S_2\} &= \frac{1}{s^\alpha}[\mathcal{L}\{P - (\mu + r + K_1X)S_1 + \tau R_1\}] \\
\mathcal{L}\{S_{v2}\} &= \frac{1}{s^\alpha}[\mathcal{L}\{rS_1 - (\mu + \lambda + K_2Z)S_{v1}\}] \\
\mathcal{L}\{S_{u2}\} &= \frac{1}{s^\alpha}[\mathcal{L}\{K_1XS_1 - (\mu + \sigma + K_3Y)S_{u1}\} \\
\mathcal{L}\{S_{vc2}\} &= \frac{1}{s^\alpha}[\mathcal{L}\left\{\lambda S_{v1} - [\mu S_{vc1} + \frac{1}{N}(m_1A_1 + m_2B_1 + m_3F_1 + m_4G_1)]\right\}] \\
\mathcal{L}\{S_{vn2}\} &= \frac{1}{s^\alpha}[\mathcal{L}\left\{K_2ZS_{v1} - [\mu S_{vn1} + \frac{1}{N}(n_1H_1 + n_2L_1 + n_3Q_1 + n_4T_1)]\right\}] \\
\mathcal{L}\{S_{uc2}\} &= \frac{1}{s^\alpha}[\mathcal{L}\left\{\sigma S_{u1} - [\mu S_{uc1} + \frac{1}{N}(e_1U_1 + e_2V_1 + e_3W_1 + e_4X_1)]\right\}] \\
\mathcal{L}\{S_{un2}\} &= \frac{1}{s^\alpha}[\mathcal{L}\left\{K_3YS_{u1} - [\mu S_{un1} + \frac{1}{N}(t_1Y_1 + t_2Z_1 + t_3N_1 + t_4M_1)]\right\}] \\
\mathcal{L}\{E_2\} &= \frac{1}{s^\alpha}[\mathcal{L}\{\frac{1}{N}[m_1A_1 + m_2B_1 + m_3F_1 + m_4G_1 + \\
&\quad n_1H_1 + n_2L_1 + n_3Q_1 + n_4T_1 + e_1U_1 + e_2V_1 + e_3W_1 + e_4X_1 + \\
&\quad t_1Y_1 + t_2Z_1 + t_3N_1 + t_4M_1] - (\mu + \alpha_1 + \alpha_2)E_1\}] \\
\mathcal{L}\{E_{Q2}\} &= \frac{1}{s^\alpha}[\mathcal{L}\{\alpha_1E_1 - (\mu + \theta_1)E_{Q1}\}] \\
\mathcal{L}\{E_{T2}\} &= \frac{1}{s^\alpha}[\mathcal{L}\{\alpha_2E_1 - (\mu + \theta_2 + \rho)E_{T1}\}] \\
\mathcal{L}\{I_{T2}\} &= \frac{1}{s^\alpha}[\mathcal{L}\{C_1E_{Q1} + C_2E_{T1} - (d_1 + J_2)I_{T1}\}] \\
\mathcal{L}\{I_{i2}\} &= \frac{1}{s^\alpha}[\mathcal{L}\{C_2E_{Q1} + C_4E_{T1} - (d_2 + J_1)I_{i1}\}] \\
\mathcal{L}\{I_{N2}\} &= \frac{1}{s^\alpha}[\mathcal{L}\{C_5E_{Q1} + C_6E_{T1} - d_3I_{N1}\}] \\
\mathcal{L}\{R_2\} &= \frac{1}{s^\alpha}[\mathcal{L}\{J_1I_{i1} + J_2I_{T1} + \rho E_{T1} - \tau R_1\}] \\
\mathcal{L}\{D_{u2}\} &= \frac{1}{s^\alpha}[\mathcal{L}\{d_3I_{N1} + d_2I_{i1} + d_1I_{T1}\}] \\
\mathcal{L}\{S_{r2}\} &= \frac{1}{s^\alpha}[\mathcal{L}\{\Lambda - \mu_rS_{r1}\}] \\
\mathcal{L}\{E_{r2}\} &= \frac{1}{s^\alpha}[\mathcal{L}\{\omega S_{r1} - \mu_rE_{r1}\}] \\
\mathcal{L}\{I_{r2}\} &= \frac{1}{s^\alpha}[\mathcal{L}\{\phi E_{r1} - d_4I_{r1}\}]
\end{aligned}
\right\} \quad (14)
$$

$$\left.\begin{aligned}
\mathcal{L}\{S_{n+1}\} &= \tfrac{1}{s^{\alpha}}[\,\mathcal{L}\{\mathrm{P} - (\mu + \mathrm{r} + \mathrm{K_1X})S_n + \tau R_n\}] \\
\mathcal{L}\{S_{v(n+1)}\} &= \tfrac{1}{s^{\alpha}}[\,\mathcal{L}\{\mathrm{r}S_n - (\mu + \lambda + \mathrm{K_2Z})\mathrm{S_{vn}}\}] \\
\mathcal{L}\{S_{u(n+1)}\} &= \tfrac{1}{s^{\alpha}}[\mathcal{L}\{\mathrm{K_1X}S_n - (\mu + \sigma + \mathrm{K_3Y})\mathrm{S_{un}}\} \\
\mathcal{L}\{S_{vc(n+1)}\} &= \tfrac{1}{s^{\alpha}}[\,\mathcal{L}\left\{\lambda \mathrm{S_{v_n}} - [\mu S_{vc_n} + \tfrac{1}{N}(m_1A_n + m_2B_n + m_3F_n + m_4G_n)]\right\}] \\
\mathcal{L}\{S_{vn(n+1)}\} &= \tfrac{1}{s^{\alpha}}[\,\mathcal{L}\left\{\mathrm{K_2Z}\mathrm{S_{v_n}} - [\mu S_{vn_n} + \tfrac{1}{N}(n_1H_n + n_2L_n + n_3Q_n + n_4T_n)]\right\}] \\
\mathcal{L}\{S_{vn(n+1)}\} &= \tfrac{1}{s^{\alpha}}[\,\mathcal{L}\left\{\mathrm{K_2Z}\mathrm{S_{v_n}} - [\mu S_{vn_n} + \tfrac{1}{N}(n_1H_n + n_2L_n + n_3Q_n + n_4T_n)]\right\}] \\
\mathcal{L}\{S_{uc(n+1)}\} &= \tfrac{1}{s^{\alpha}}[\,\mathcal{L}\left\{\sigma \mathrm{S_{u_n}} - [\mu S_{uc_n} + \tfrac{1}{N}(e_1U_n + e_2V_n + e_3W_n + e_4X_n)]\right\}] \\
\mathcal{L}\{S_{un(n+1)}\} &= \tfrac{1}{s^{\alpha}}[\mathcal{L}\left\{\mathrm{K_3Y}\mathrm{S_{u_n}} - [\mu S_{un_n} + \tfrac{1}{N}(t_1Y_n + t_2Z_n + t_3I_rN_n + t_4M_n)]\right\}] \\
\mathcal{L}\{E_{(n+1)}\} &= \tfrac{1}{s^{\alpha}}[\mathcal{L}\{\tfrac{1}{N}[m_1A_n + m_2B_n + m_3F_n + m_4G_n + \\
&\quad n_1H_n + n_2L_n + n_3Q_n + n_4T_n + e_1U_n + e_2V_n + e_3W_n + e_4X_n + \\
&\quad t_1Y_n + t_2Z_n + t_3N_n + t_4M_n] - (\mu + \alpha_1 + \alpha_2)\,E_n\}] \\
\mathcal{L}\{E_{Q(n+1)}\} &= \tfrac{1}{s^{\alpha}}[\,\mathcal{L}\{\,\alpha_1E_n - (\mu + \theta_1)\mathrm{E_{Qn}}\}] \\
\mathcal{L}\{E_{T(n+1)}\} &= \tfrac{1}{s^{\alpha}}[\,\mathcal{L}\{\,\alpha_2E_n - (\mu + \theta_2 + \rho)\mathrm{E_{Tn}}\}] \\
\mathcal{L}\{I_{T(n+1)}\} &= \tfrac{1}{s^{\alpha}}[\,\mathcal{L}\{\,\mathrm{C_1E_{Qn}} + \mathrm{C_2E_{Tn}} - (\mathrm{d_1} + \mathrm{J_2})\mathrm{I_{Tn}}\}] \\
\mathcal{L}\{I_{i(n+1)}\} &= \tfrac{1}{s^{\alpha}}[\mathcal{L}\{\,\mathrm{C_2E_{Qn}} + \mathrm{C_4E_{Tn}} - (\mathrm{d_2} + \mathrm{J_1})\mathrm{I_{in}}\}] \\
\mathcal{L}\{I_{N(n+1)}\} &= \tfrac{1}{s^{\alpha}}[\mathcal{L}\{\,\mathrm{C_5E_{Qn}} + \mathrm{C_6E_{Tn}} - \mathrm{d_3I_{Nn}}\}] \\
\mathcal{L}\{R_{n+1}\} &= \tfrac{1}{s^{\alpha}}[\mathcal{L}\{\,\mathrm{J_1I_{in}} + \mathrm{J_2I_{Tn}} + \rho\mathrm{E_{Tn}} - \tau R_n\}] \\
\mathcal{L}\{D_{u(n+1)}\} &= \tfrac{1}{s^{\alpha}}[\mathcal{L}\{\,\mathrm{d_3I_{Nn}} + \mathrm{d_2I_{in}} + \mathrm{d_1I_{Tn}}\}] \\
\mathcal{L}\{S_{r(n+1)}\} &= \tfrac{1}{s^{\alpha}}[\mathcal{L}\{\,\Lambda - \mu_r\mathrm{S_{rn}}\}] \\
\mathcal{L}\{E_{r(n+1)}\} &= \tfrac{1}{s^{\alpha}}[\mathcal{L}\{\,\omega\mathrm{S_{rn}} - \mu_r\mathrm{E_{rn}}\}] \\
\mathcal{L}\{I_{r(n+1)}\} &= \tfrac{1}{s^{\alpha}}[\mathcal{L}\{\,\phi\mathrm{E_{rn}} - \mathrm{d_4I_{rn}}\}]
\end{aligned}\right\} \quad (15)$$

Evaluating the Laplace transforms on the right hand side of (13), (14) and (15) we have the following;

$$\left.\begin{aligned}
\mathcal{L}\{S_1\} &= \{\mathrm{P} - (\mu + \mathrm{r} + \mathrm{K_1X})S_0 + \tau R_0\}\tfrac{1}{s^{\alpha+1}} \\
\mathcal{L}\{S_{v1}\} &= \{\,\mathrm{r}S_0 - (\mu + \lambda + \mathrm{K_2Z})\mathrm{S_{v0}}\}\tfrac{1}{s^{\alpha+1}} \\
\mathcal{L}\{S_{u1}\} &= \{\mathrm{K_1X}S_0 - (\mu + \sigma + \mathrm{K_3Y})\mathrm{S_{u0}}\}\tfrac{1}{s^{\alpha+1}} \\
\mathcal{L}\{S_{vc1}\} &= \left\{\lambda \mathrm{S_{v0}} - [\mu S_{vc0} + \tfrac{1}{N}(m_1A_0 + m_2B_0 + m_3F_0 + m_4G_0)]\right\}\tfrac{1}{s^{\alpha+1}} \\
\mathcal{L}\{S_{vn1}\} &= \left\{\mathrm{K_2Z}\mathrm{S_{v0}} - [\mu S_{vn0} + \tfrac{1}{N}(n_1H_0 + n_2L_0 + n_3Q_0 + n_4T_0)]\right\}\tfrac{1}{s^{\alpha+1}} \\
\mathcal{L}\{S_{uc1}\} &= \left\{\sigma \mathrm{S_{u0}} - [\mu S_{uc0} + \tfrac{1}{N}(e_1U_0 + e_2V_0 + e_3W_0 + e_4X_0)]\right\}\tfrac{1}{s^{\alpha+1}} \\
\mathcal{L}\{S_{un1}\} &= \left\{\mathrm{K_3Y}\mathrm{S_{u0}} - [\mu S_{un0} + \tfrac{1}{N}(t_1Y_0 + t_2Z_0 + t_3I_rN_0 + t_4M_0)]\right\}\tfrac{1}{s^{\alpha+1}} \\
\mathcal{L}\{E_1\} &= \{\tfrac{1}{N}[m_1A_0 + m_2B_0 + m_3F_0 + m_4G_0 + \\
&\quad n_1H_0 + n_2L_0 + n_3Q_0 + n_4T_0 + e_1U_0 + e_2V_0 + e_3W_0 + e_4X_0 + \\
&\quad t_1Y_0 + t_2Z_0 + t_3N_0 + t_4M_0] - (\mu + \alpha_1 + \alpha_2)\,E_0\}\tfrac{1}{s^{\alpha+1}} \\
\mathcal{L}\{E_{Q1}\} &= \{\,\alpha_1E_0 - (\mu + \theta_1)\mathrm{E_{Q0}}\}\tfrac{1}{s^{\alpha+1}} \\
\mathcal{L}\{E_{T1}\} &= \{\,\alpha_2E_0 - (\mu + \theta_2 + \rho)\mathrm{E_{T0}}\}\tfrac{1}{s^{\alpha+1}} \\
\mathcal{L}\{I_{T1}\} &= \{\,\mathrm{C_1E_{Q0}} + \mathrm{C_2E_{T0}} - (\mathrm{d_1} + \mathrm{J_2})\mathrm{I_{T0}}\}\tfrac{1}{s^{\alpha+1}} \\
\mathcal{L}\{I_{i1}\} &= \{\,\mathrm{C_2E_{Q0}} + \mathrm{C_4E_{T0}} - (\mathrm{d_2} + \mathrm{J_1})\mathrm{I_{i0}}\}\tfrac{1}{s^{\alpha+1}} \\
\mathcal{L}\{I_{N1}\} &= \{\,\mathrm{C_5E_{Q0}} + \mathrm{C_6E_{T0}} - \mathrm{d_3I_{N0}}\}\tfrac{1}{s^{\alpha+1}} \\
\mathcal{L}\{R_1\} &= \{\,\mathrm{J_1I_{i0}} + \mathrm{J_2I_{T0}} + \rho\mathrm{E_{T0}} - \tau R_0\}\tfrac{1}{s^{\alpha+1}} \\
\mathcal{L}\{D_{u1}\} &= \{\,\mathrm{d_3I_{N0}} + \mathrm{d_2I_{i0}} + \mathrm{d_1I_{T0}}\}\tfrac{1}{s^{\alpha+1}} \\
\mathcal{L}\{S_{r1}\} &= \{\,\Lambda - \mu_r\mathrm{S_{r0}}\}\tfrac{1}{s^{\alpha+1}} \\
\mathcal{L}\{E_{r1}\} &= \{\,\omega\mathrm{S_{r0}} - \mu_r\mathrm{E_{r0}}\}\tfrac{1}{s^{\alpha+1}} \\
\mathcal{L}\{I_{r1}\} &= \{\,\phi\mathrm{E_{r0}} - \mathrm{d_4I_{r0}}\}\tfrac{1}{s^{\alpha+1}}
\end{aligned}\right\} \quad (16)$$

$$\left.\begin{aligned}
\mathcal{L}\{S_2\} &= \{P - (\mu + r + K_1X)S_1 + \tau R_1\}\frac{1}{S^{\alpha+1}}\\
\mathcal{L}\{S_{v2}\} &= \{rS_1 - (\mu + \lambda + K_2Z)S_{v1}\}\frac{1}{S^{\alpha+1}}\\
\mathcal{L}\{S_{u2}\} &= \{K_1XS_1 - (\mu + \sigma + K_3Y)S_{u1}\}\frac{1}{S^{\alpha+1}}\\
\mathcal{L}\{S_{vc2}\} &= \left\{\lambda S_{v1} - \left[\mu S_{vc1} + \frac{1}{N}(m_1A_1 + m_2B_1 + m_3F_1 + m_4G_1)\right]\right\}\frac{1}{S^{\alpha+1}}\\
\mathcal{L}\{S_{vn2}\} &= \left\{K_2ZS_{v1} - \left[\mu S_{vn1} + \frac{1}{N}(n_1H_1 + n_2L_1 + n_3Q_1 + n_4T_1)\right]\right\}\frac{1}{S^{\alpha+1}}\\
\mathcal{L}\{S_{uc2}\} &= \left\{\sigma S_{u1} - \left[\mu S_{uc1} + \frac{1}{N}(e_1U_1 + e_2V_1 + e_3W_1 + e_4X_1)\right]\right\}\frac{1}{S^{\alpha+1}}\\
\mathcal{L}\{S_{un2}\} &= \left\{K_3YS_{u1} - \left[\mu S_{un1} + \frac{1}{N}(t_1Y_1 + t_2Z_1 + t_3N_1 + t_4M_1)\right]\right\}\frac{1}{S^{\alpha+1}}\\
\mathcal{L}\{E_2\} &= \{\frac{1}{N}[m_1A_1 + m_2B_1 + m_3F_1 + m_4G_1 +\\
&\quad n_1H_1 + n_2L_1 + n_3Q_1 + n_4T_1 + e_1U_1 + e_2V_1 + e_3W_1 + e_4X_1 +\\
&\quad t_1Y_1 + t_2Z_1 + t_3N_1 + t_4M_1] - (\mu + \alpha_1 + \alpha_2)E_0\}\frac{1}{S^{\alpha+1}}\\
\mathcal{L}\{E_{Q2}\} &= \{\alpha_1E_1 - (\mu + \theta_1)E_{Q1}\}\frac{1}{S^{\alpha+1}}\\
\mathcal{L}\{E_{T2}\} &= \{\alpha_2E_1 - (\mu + \theta_2 + \rho)E_{T1}\}\frac{1}{S^{\alpha+1}}\\
\mathcal{L}\{I_{T2}\} &= \{C_1E_{Q1} + C_2E_{T1} - (d_1 + J_2)I_{T1}\}\frac{1}{S^{\alpha+1}}\\
\mathcal{L}\{I_{i2}\} &= \{C_2E_{Q1} + C_4E_{T1} - (d_2 + J_1)I_{i1}\}\frac{1}{S^{\alpha+1}}\\
\mathcal{L}\{I_{N2}\} &= \{C_5E_{Q1} + C_6E_{T1} - d_3I_{N1}\}\frac{1}{S^{\alpha+1}}\\
\mathcal{L}\{R_2\} &= \{J_1I_{i1} + J_2I_{T1} + \rho E_{T1} - \tau R_1\}\frac{1}{S^{\alpha+1}}\\
\mathcal{L}\{D_{u2}\} &= \{d_3I_{N1} + d_2I_{i1} + d_1I_{T1}\}\frac{1}{S^{\alpha+1}}\\
\mathcal{L}\{S_{r2}\} &= \{\Lambda - \mu_rS_{r1}\}\frac{1}{S^{\alpha+1}}\\
\mathcal{L}\{E_{r2}\} &= \{\omega S_{r1} - \mu_rE_{r1}\}\frac{1}{S^{\alpha+1}}\\
\mathcal{L}\{I_{r2}\} &= \{\phi E_{r1} - d_4I_{r1}\}\frac{1}{S^{\alpha+1}}
\end{aligned}\right\} \quad (17)$$

$$\left.\begin{aligned}
\mathcal{L}\{S_{n+1}\} &= \{P - (\mu + r + K_1X)S_n + \tau R_n\}\frac{1}{S^{\alpha+1}}\\
\mathcal{L}\{S_{v(n+1)}\} &= \{rS_n - (\mu + \lambda + K_2Z)S_{vn}\}\frac{1}{S^{\alpha+1}}\\
\mathcal{L}\{S_{u(n+1)}\} &= \{K_1XS_n - (\mu + \sigma + K_3Y)S_{un}\}\frac{1}{S^{\alpha+1}}\\
\mathcal{L}\{S_{vc(n+1)}\} &= \left\{\lambda S_{v_n} - \left[\mu S_{vc_n} + \frac{1}{N}(m_1A_n + m_2B_n + m_3F_n + m_4G_n)\right]\right\}\frac{1}{S^{\alpha+1}}\\
\mathcal{L}\{S_{vn(n+1)}\} &= \left\{K_2ZS_{v_n} - \left[\mu S_{vn_n} + \frac{1}{N}(n_1H_n + n_2L_n + n_3Q_n + n_4T_n)\right]\right\}\frac{1}{S^{\alpha+1}}\\
\mathcal{L}\{S_{uc(n+1)}\} &= \left\{\sigma S_{u_n} - \left[\mu S_{uc_n} + \frac{1}{N}(e_1U_n + e_2V_n + e_3W_n + e_4X_n)\right]\right\}\frac{1}{S^{\alpha+1}}\\
\mathcal{L}\{S_{un(n+1)}\} &= \left\{K_3YS_{u_n} - \left[\mu S_{un_n} + \frac{1}{N}(t_1Y_n + t_2Z_n + t_3N_n + t_4M_n)\right]\right\}\frac{1}{S^{\alpha+1}}\\
\mathcal{L}\{E_{(n+1)}\} &= \{\frac{1}{N}[m_1A_n + m_2B_n + m_3F_n + m_4G_n +\\
&\quad n_1H_n + n_2L_n + n_3Q_n + n_4T_n + e_1U_n + e_2V_n + e_3W_n + e_4X_n +\\
&\quad t_1Y_n + t_2Z_n + t_3N_n + t_4M_n] - (\mu + \alpha_1 + \alpha_2)E_n\}\frac{1}{S^{\alpha+1}}\\
\mathcal{L}\{E_{Q(n+1)}\} &= \{\alpha_1E_n - (\mu + \theta_1)E_{Qn}\}\frac{1}{S^{\alpha+1}}\\
\mathcal{L}\{E_{T(n+1)}\} &= \{\alpha_2E_n - (\mu + \theta_2 + \rho)E_{Tn}\}\frac{1}{S^{\alpha+1}}\\
\mathcal{L}\{I_{T(n+1)}\} &= \{C_1E_{Qn} + C_2E_{Tn} - (d_1 + J_2)I_{Tn}\}\frac{1}{S^{\alpha+1}}\\
\mathcal{L}\{I_{i(n+1)}\} &= \{C_2E_{Qn} + C_4E_{Tn} - (d_2 + J_1)I_{in}\}\frac{1}{S^{\alpha+1}}\\
\mathcal{L}\{I_{N(n+1)}\} &= \{C_5E_{Qn} + C_6E_{Tn} - d_3I_{Nn}\}\frac{1}{S^{\alpha+1}}\\
\mathcal{L}\{R_{n+1}\} &= \{J_1I_{in} + J_2I_{Tn} + \rho E_{Tn} - \tau R_n\}\frac{1}{S^{\alpha+1}}\\
\mathcal{L}\{D_{u(n+1)}\} &= \{d_3I_{Nn} + d_2I_{in} + d_1I_{Tn}\}\frac{1}{S^{\alpha+1}}\\
\mathcal{L}\{S_{r(n+1)}\} &= \{\Lambda - \mu_rS_{rn}\}\frac{1}{S^{\alpha+1}}\\
\mathcal{L}\{E_{r(n+1)}\} &= \{\omega S_{rn} - \mu_rE_{rn}\}\frac{1}{S^{\alpha+1}}\\
\mathcal{L}\{I_{r(n+1)}\} &= \{\phi E_{rn} - d_4I_{rn}\}\frac{1}{S^{\alpha+1}}
\end{aligned}\right\} \quad (18)$$

Taking the Laplace inverse transform of (15), (16) and (17) and applying the initial conditions (2) we have;

$$\left.\begin{aligned}
&S_0 = g_1, S_{v0} = g_2, S_{u0} = g_3\ S_{vc0} = g_4,\\
&S_{vn0} = g_5, S_{uc0} = g_6, S_{un0} = g_7, E_0 = g_8, E_{Q0} = g_9,\\
&E_{T0} = g_{10}, I_{T0}(0) = g_{11}, I_{i0} = g_{12}, I_{N0} = g_{13}, R_0 = g_{14},\\
&D_{u0} = g_{15}, \quad S_{r0} = g_{16}, \quad E_{r0} = g_{17}\ and\ I_{r0} = g_{18}
\end{aligned}\right\} \quad (19)$$

$$
\left.
\begin{aligned}
S_1 &= \{P - (\mu + r + K_1X)g_1 + \tau g_{14}\}\frac{t^\alpha}{\Gamma(\alpha+1)} \\
S_{v1} &= \{ rg_1 - (\mu + \lambda + K_2Z)g_2\}\frac{t^\alpha}{\Gamma(\alpha+1)} \\
S_{u1} &= \{K_1Xg_1 - (\mu + \sigma + K_3Y)g_3\}\frac{t^\alpha}{\Gamma(\alpha+1)} \\
S_{vc1} &= \left\{\lambda g_2 - [\mu g_4 + \frac{1}{N}(m_1g_4g_{13} + m_2g_4g_{11} + m_3g_4g_{18} + m_4g_4g_{15})]\right\}\frac{t^\alpha}{\Gamma(\alpha+1)} \\
S_{vn1} &= \left\{K_2Zg_2 - [\mu g_5 + \frac{1}{N}(n_1g_5g_{13} + n_2g_5g_{11} + n_3g_5g_{18} + n_4g_5g_{15})]\right\}\frac{t^\alpha}{\Gamma(\alpha+1)} \\
S_{uc1} &= \left\{\sigma g_3 - [\mu g_6 + \frac{1}{N}(e_1g_6g_{13} + e_2g_6g_{11} + e_3g_6g_{18} + e_4g_6g_{15})]\right\}\frac{t^\alpha}{\Gamma(\alpha+1)} \\
S_{un1} &= \left\{K_3Yg_3 - [\mu g_7 + \frac{1}{N}(t_1g_7g_{13} + t_2g_7g_{11} + t_3g_7g_{18} + t_4g_7g_{15})]\right\}\frac{t^\alpha}{\Gamma(\alpha+1)} \\
E_1 &= \{\frac{1}{N}[m_1g_4g_{13} + m_2g_4g_{11} + m_3g_4g_{18} + m_4g_4g_{15} + \\
&\quad n_1g_5g_{13} + n_2g_5g_{11} + n_3g_5g_{18} + n_4g_5g_{15} + e_1g_6g_{13} + e_2g_6g_{11} + e_3g_6g_{18} + \\
&\quad e_4g_6g_{15} + t_1g_7g_{13} + t_2g_7g_{11} + t_3g_6g_{18} + t_4g_7g_{15}] - (\mu + \alpha_1 + \alpha_2)\, g_8\}\frac{t^\alpha}{\Gamma(\alpha+1)} \\
E_{Q1} &= \{ \alpha_1g_8 - (\mu + \theta_1)g_9\}\frac{t^\alpha}{\Gamma(\alpha+1)} \\
E_{T1} &= \{ \alpha_2g_8 - (\mu + \theta_2 + \rho)g_{10}\}\frac{t^\alpha}{\Gamma(\alpha+1)} \\
I_{T1} &= \{ C_1g_9 + C_2g_{10} - (d_1 + J_2)g_{11}\}\frac{t^\alpha}{\Gamma(\alpha+1)} \\
I_{i1} &= \{ C_2g_9 + C_4g_{10} - (d_2 + J_1)g_{12}\}\frac{t^\alpha}{\Gamma(\alpha+1)} \\
I_{N1} &= \{ C_5g_9 + C_6g_{10} - d_3g_{13}\}\frac{t^\alpha}{\Gamma(\alpha+1)} \\
R_1 &= \{ J_1g_{12} + J_2g_{11} + \rho g_{10} - \tau g_{14}\}\frac{t^\alpha}{\Gamma(\alpha+1)} \\
D_{u1} &= \{ d_3g_{13} + d_2g_{12} + d_1g_{11}\}\frac{t^\alpha}{\Gamma(\alpha+1)} \\
S_{r1} &= \{ \Lambda - \mu_rg_{16}\}\frac{t^\alpha}{\Gamma(\alpha+1)} \\
E_{r1} &= \{ \omega g_{16} - \mu_rg_{17}\}\frac{t^\alpha}{\Gamma(\alpha+1)} \\
I_{r1} &= \{ \phi g_{17} - d_4g_{18}\}\frac{t^\alpha}{\Gamma(\alpha+1)}
\end{aligned}
\right\} \quad (20)
$$

$$
\left.
\begin{aligned}
S_2 &= \{P - (\mu + r + K_1X)S_1 + \tau R_1\}\frac{t^\alpha}{\Gamma(\alpha+1)} \\
S_{v2} &= \{ rS_1 - (\mu + \lambda + K_2Z)S_{v1}\}\frac{t^\alpha}{\Gamma(\alpha+1)} \\
S_{u2} &= \{K_1XS_1 - (\mu + \sigma + K_3Y)S_{u1}\}\frac{t^\alpha}{\Gamma(\alpha+1)} \\
S_{vc2} &= \left\{\lambda S_{v1} - [\mu S_{vc_1} + \frac{1}{N}(m_1A_1 + m_2B_1 + m_3F_1 + m_4G_1)]\right\}\frac{t^\alpha}{\Gamma(\alpha+1)} \\
S_{vn2} &= \left\{K_2ZS_{v1} - [\mu S_{vn_1} + \frac{1}{N}(n_1H_1 + n_2L_1 + n_3Q_1 + n_4T_1)]\right\}\frac{t^\alpha}{\Gamma(\alpha+1)} \\
S_{uc2} &= \left\{\sigma S_{u1} - [\mu S_{uc_1} + \frac{1}{N}(e_1U_1 + e_2V_1 + e_3W_1 + e_4X_1)]\right\}\frac{t^\alpha}{\Gamma(\alpha+1)} \\
S_{un2} &= \left\{K_3YS_{u1} - [\mu S_{un_1} + \frac{1}{N}(t_1Y_1 + t_2Z_1 + t_3N_1 + t_4M_1)]\right\}\frac{t^\alpha}{\Gamma(\alpha+1)} \\
E_2 &= \{\frac{1}{N}[m_1A_1 + m_2B_1 + m_3F_1 + m_4G_1 + n_1H_1 + n_2L_1 + \\
&\quad n_3Q_1 + n_4T_1 + e_1U_1 + e_2V_1 + e_3W_1 + e_4X_1 + t_1Y_1 + t_2Z_1 + \\
&\quad t_3N_1 + t_4M_1] - (\mu + \alpha_1 + \alpha_2)\, E_1\}\frac{t^\alpha}{\Gamma(\alpha+1)} \\
E_{Q2} &= \{ \alpha_1E_1 - (\mu + \theta_1)E_{Q1}\}\frac{t^\alpha}{\Gamma(\alpha+1)} \\
E_{T2} &= \{ \alpha_2E_1 - (\mu + \theta_2 + \rho)E_{T1}\}\frac{t^\alpha}{\Gamma(\alpha+1)} \\
I_{T2} &= \{ C_1E_{Q1} + C_2E_{T1} - (d_1 + J_2)I_{T1}\}\frac{t^\alpha}{\Gamma(\alpha+1)} \\
I_{i2} &= \{ C_2E_{Q1} + C_4E_{T1} - (d_2 + J_1)I_{i1}\}\frac{t^\alpha}{\Gamma(\alpha+1)} \\
I_{N2} &= \{ C_5E_{Q1} + C_6E_{T1} - d_3I_{N1}\}\frac{t^\alpha}{\Gamma(\alpha+1)} \\
R_2 &= \{ J_1I_{i1} + J_2I_{T1} + \rho E_{T1} - \tau R_1\}\frac{t^\alpha}{\Gamma(\alpha+1)} \\
D_{u2} &= \{ d_3I_{N1} + d_2I_{i1} + d_1I_{T1}\}\frac{t^\alpha}{\Gamma(\alpha+1)} \\
S_{r2} &= \{ \Lambda - \mu_rS_{r1}\}\frac{t^\alpha}{\Gamma(\alpha+1)} \\
E_{r2} &= \{ \omega S_{r1} - \mu_rE_{r1}\}\frac{t^\alpha}{\Gamma(\alpha+1)} \\
I_{r2} &= \{ \phi E_{r1} - d_4I_{r1}\}\frac{t^\alpha}{\Gamma(\alpha+1)}
\end{aligned}
\right\} \quad (21)
$$

We can calculate the other terms in the same way.

Finally, we get the EVD fractional solution in the form of infinite series (7) as follows;

$$\left.\begin{aligned} S(t) &= S_0 + S_1 + S_2 + \dots \\ &= g_1 + \{P - (\mu + r + K_1X)g_1 + \tau g_{14}\}\frac{t^\alpha}{\Gamma(\alpha+1)} \\ &+ \{P - (\mu + r + K_1X)S_1 + \tau R_1\}\frac{t^\alpha}{\Gamma(\alpha+1)} + \dots \end{aligned}\right\} \quad (22)$$

$$\left.\begin{aligned} S_v(t) &= S_{v0} + S_{v1} + S_{v2} + \dots \\ &= g_2 + \{rg_1 - (\mu + \lambda + K_2Z)g_2\}\frac{t^\alpha}{\Gamma(\alpha+1)} \\ &+ \{rS_1 - (\mu + \lambda + K_2Z)S_{v1}\}\frac{t^\alpha}{\Gamma(\alpha+1)} + \dots \end{aligned}\right\} \quad (23)$$

$$\left.\begin{aligned} S_u(t) &= S_{u0} + S_{u1} + S_{u2} + \dots \\ &= g_3 + \{K_1Xg_1 - (\mu + \sigma + K_3Y)g_3\}\frac{t^\alpha}{\Gamma(\alpha+1)} \\ &+ \{K_1XS_1 - (\mu + \sigma + K_3Y)S_{u1}\}\frac{t^\alpha}{\Gamma(\alpha+1)} + \dots \end{aligned}\right\} \quad (24)$$

$$\left.\begin{aligned} S_{vc}(t) &= S_{vc0} + S_{vc1} + S_{vc2} + \dots \\ &= g_4 + \left\{\lambda g_2 - \left[\mu g_4 + \frac{1}{N}(m_1g_4g_{13} + m_2g_4g_{11} + m_3g_4g_{18} + m_4g_4g_{15})\right]\right\}\frac{t^\alpha}{\Gamma(\alpha+1)} \\ &+ \left\{\lambda S_{v1} - \left[\mu S_{vc} + \frac{1}{N}(m_1A_1 + m_2B_1 + m_3F_1 + m_4G_1)\right]\right\}\frac{t^\alpha}{\Gamma(\alpha+1)} + \dots \end{aligned}\right\} \quad (25)$$

$$\left.\begin{aligned} S_{vn}(t) &= S_{vn0} + S_{vn1} + S_{vn} + \dots \\ &= g_5 + \left\{K_2Zg_2 - \left[\mu g_5 + \frac{1}{N}(n_1g_5g_{13} + n_2g_5g_{11} + n_3g_5g_{18} + n_4g_5g_{15})\right]\right\}\frac{t^\alpha}{\Gamma(\alpha+1)} \\ &+ \left\{K_2ZS_{v1} - \left[\mu S_{vn1} + \frac{1}{N}(n_1H_1 + n_2L_1 + n_3Q_1 + n_4T_1)\right]\right\}\frac{t^\alpha}{\Gamma(\alpha+1)} + \dots \end{aligned}\right\} \quad (26)$$

$$\left.\begin{aligned} S_{uc}(t) &= S_{uc0} + S_{uc1} + S_{uc2} + \dots \\ &= g_6 + \left\{\sigma g_3 - \left[\mu g_6 + \frac{1}{N}(e_1g_6g_{13} + e_2g_6g_{11} + e_3g_6g_{18} + e_4g_6g_{15})\right]\right\}\frac{t^\alpha}{\Gamma(\alpha+1)} \\ &+ \left\{\sigma S_{u1} - \left[\mu S_{uc1} + \frac{1}{N}(e_1U_1 + e_2V_1 + e_3W_1 + e_4X_1)\right]\right\}\frac{t^\alpha}{\Gamma(\alpha+1)} + \dots \end{aligned}\right\} \quad (27)$$

$$\left.\begin{aligned} S_{un}(t) &= S_{un0} + S_{un} + S_{un2} + \dots \\ &= g_7 + \left\{K_3Yg_3 - \left[\mu g_7 + \frac{1}{N}(t_1g_7g_{13} + t_2g_7g_{11} + t_3g_7g_{18} + t_4g_7g_{15})\right]\right\}\frac{t^\alpha}{\Gamma(\alpha+1)} \\ &+ \left\{K_3YS_{u1} - \left[\mu S_{un1} + \frac{1}{N}(t_1Y_1 + t_2Z_1 + t_3N_1 + t_4M_1)\right]\right\}\frac{t^\alpha}{\Gamma(\alpha+1)} + \dots \end{aligned}\right\} \quad (28)$$

$$\left.\begin{aligned} E(t) &= E_0 + E_1 + E_2 + \dots \\ &= g_8 + \{\frac{1}{N}[m_1g_4g_{13} + m_2g_4g_{11} + m_3g_4g_{18} + m_4g_4g_{15} + n_1g_5g_{13} + n_2g_5g_{11} + n_3g_5g_{18} + \\ &n_4g_5g_{15} + e_1g_6g_{13} + e_2g_6g_{11} + e_3g_6g_{18} + e_4g_6g_{15} + t_1g_7g_{13} + t_2g_7g_{11} + t_3g_6g_{18} + t_4g_7g_{15}] - \\ &(\mu + \alpha_1 + \alpha_2)\, g_8\}\frac{t^\alpha}{\Gamma(\alpha+1)} + \{\frac{1}{N}[m_1A_1 + m_2B_1 + m_3F_1 + m_4G_1 + n_1H_1 + n_2L_1 + \\ &n_3Q_1 + n_4T_1 + e_1U_1 + e_2V_1 + e_3W_1 + e_4X_1 + t_1Y_1 + t_2Z_1 + \\ &t_3N_1 + t_4M_1] - (\mu + \alpha_1 + \alpha_2)\, E_1\}\frac{t^\alpha}{\Gamma(\alpha+1)} + \dots \end{aligned}\right\} \quad (29)$$

$$\left.\begin{aligned} E_Q(t) &= E_{Q0} + E_{Q1} + E_{Q2} + \dots \\ &= g_9 + \{\alpha_1g_8 - (\mu + \theta_1)g_9\}\frac{t^\alpha}{\Gamma(\alpha+1)} + \{\alpha_1E_1 - (\mu + \theta_1)E_{Q1}\}\frac{t^\alpha}{\Gamma(\alpha+1)} + \dots \end{aligned}\right\} \quad (30)$$

$$\left.\begin{aligned} E_T(t) &= E_{T0} + E_{T1} + E_{T2} + \dots \\ &= g_{10} + \{\alpha_2g_8 - (\mu + \theta_2 + \rho)g_{10}\}\frac{t^\alpha}{\Gamma(\alpha+1)} + \{\alpha_2E_1 - (\mu + \theta_2 + \rho)E_{T1}\}\frac{t^\alpha}{\Gamma(\alpha+1)} + \dots \end{aligned}\right\} \quad (31)$$

$$\left.\begin{aligned} I_T(t) &= I_{T0} + I_{T1} + I_{T2} + \dots \\ &= g_{11} + \{C_1g_9 + C_2g_{10} - (d_1 + J_2)g_{11}\}\frac{t^\alpha}{\Gamma(\alpha+1)} + \{C_1E_{Q1} + C_2E_{T1} - (d_1 + J_2)I_{T1}\}\frac{t^\alpha}{\Gamma(\alpha+1)} + \dots \end{aligned}\right\} \quad (32)$$

$$\left.\begin{aligned} I_i(t) &= I_{i0} + I_{i1} + I_{i2} + \dots \\ &= g_{12} + \{C_2 g_9 + C_4 g_{10} - (d_2 + J_1) g_{12}\} \frac{t^\alpha}{\Gamma(\alpha+1)} \\ &+ \{C_2 E_{Q1} + C_4 E_{T1} - (d_2 + J_1) I_{i1}\} \frac{t^\alpha}{\Gamma(\alpha+1)} + \cdots \end{aligned}\right\} \quad (33)$$

$$\left.\begin{aligned} I_N(t) &= I_{N0} + I_{N1} + I_{N2} + \cdots \\ &= g_{13} + \{C_5 g_9 + C_6 g_{10} - d_3 g_{13}\} \frac{t^\alpha}{\Gamma(\alpha+1)} + \{C_5 E_{Q1} + C_6 E_{T1} - d_3 I_{N1}\} \frac{t^\alpha}{\Gamma(\alpha+1)} + \cdots \end{aligned}\right\} \quad (34)$$

$$\left.\begin{aligned} R(t) &= R_0 + R_1 + R_2 + \dots \\ &= g_{14} + \{J_1 g_{12} + J_2 g_{11} + \rho g_{10} - \tau g_{14}\} \frac{t^\alpha}{\Gamma(\alpha+1)} + \{J_1 I_{i1} + J_2 I_{T1} + \rho E_{T1} - \tau R_1\} \frac{t^\alpha}{\Gamma(\alpha+1)} + \cdots \end{aligned}\right\} \quad (35)$$

$$\left.\begin{aligned} D_u(t) &= D_{u0} + D_{u1} + D_{u2} + \dots \\ &= g_{15} + \{d_3 g_{13} + d_2 g_{12} + d_1 g_{11}\} \frac{t^\alpha}{\Gamma(\alpha+1)} + \{d_3 I_{N1} + d_2 I_{i1} + d_1 I_{T1}\} \frac{t^\alpha}{\Gamma(\alpha+1)} + \cdots \end{aligned}\right\} \quad (36)$$

$$\left.\begin{aligned} S_r(t) &= S_{r0} + S_{r1} + S_{r2} + \dots \\ &= g_{16} + \{\Lambda - \mu_r g_{16}\} \frac{t^\alpha}{\Gamma(\alpha+1)} + \{\Lambda - \mu_r S_{r1}\} \frac{t^\alpha}{\Gamma(\alpha+1)} + \cdots \end{aligned}\right\} \quad (37)$$

$$\left.\begin{aligned} E_r(t) &= E_{r0} + E_{r1} + E_{r2} + \dots \\ &= g_{17} + \{\omega g_{16} - \mu_r g_{17}\} \frac{t^\alpha}{\Gamma(\alpha+1)} + \{\omega S_{r1} - \mu_r E_{r1}\} \frac{t^\alpha}{\Gamma(\alpha+1)} + \cdots \end{aligned}\right\} \quad (38)$$

$$\left.\begin{aligned} I_r(t) &= I_{r0} + I_{r1} + I_{r2} + \dots \\ &= g_{18} + \{\phi g_{17} - d_4 g_{18}\} \frac{t^\alpha}{\Gamma(\alpha+1)} + \{\phi E_{r1} - d_4 I_{r1}\} \frac{t^\alpha}{\Gamma(\alpha+1)} + \cdots \end{aligned}\right\} \quad (39)$$

Results and Discussion

In this section, we shall consider the numerical solution of the large compartmental EVD model.

Using Maple Soft and the values in Table 1 we obtained the solution of equations (22) - (39) as follows;

$$\left.\begin{aligned} S(t) &= 4396521 - 4.119316038\,10^6 t + 1.930306565\,10^6 t^2 + \cdots \\ S_v(t) &= 1758608 + 1.63754582610^6 t + 8.887274825\,10^5 t^2 + \cdots \\ S_u(t) &= 2637913 + 2.127107168\,10^6 t + 1.200524546\,10^6 t^2 + \cdots \\ S_{vc}(t) &= 527582 + 31633.85672t + 23929.87758t^2 + \cdots \\ S_{vn}(t) &= 1231026 - 54917.85064t + 22195.26974t^2 + \cdots \\ S_{uc}(t) &= 1582748 - 6.06783399\,10^5 t + 2.261567366t^2 + \cdots \\ S_{un}(t) &= 1055165 - 2.675265248\,10^5 t + 59916.42590t^2 + \cdots \\ E(t) &= 1.252122938\,10^6 t + 6.602270550\,10^5 t^2 + \cdots \\ E_Q(t) &= 74 - 69.06716t + 5.974857650\,10^5 t^2 + \cdots \\ E_T(t) &= 2.56686559\,10^5 t^2 + \cdots \\ I_T(t) &= 500 - 226.9290777t + 46.10402364t^2 + \cdots \\ I_i(t) &= 800 - 376.9046626t + 80.00712360t^2 + \cdots \\ I_N(t) &= 400 - 127.0771285t + 17.03932643t^2 + \cdots \\ R(t) &= 649.8800t + 173.5553478t^2 + \cdots \\ D_u(t) &= 24 + 139.7318904t + 56.97545005t^2 + \cdots \\ S_r(t) &= 6000 + 1760.00t - 35.20000000t^2 + \cdots \\ E_r(t) &= 100 + 4796.00t + 608.0800000t^2 + \cdots \\ I_r(t) &= 80.0t + 1889.960000t^2 + \cdots \end{aligned}\right\} \quad (40)$$

We plot (40) using Maple Soft and obtained Figure 1, Figure 2, Figure 3, Figure 4, Figure 5 and Figure 6.

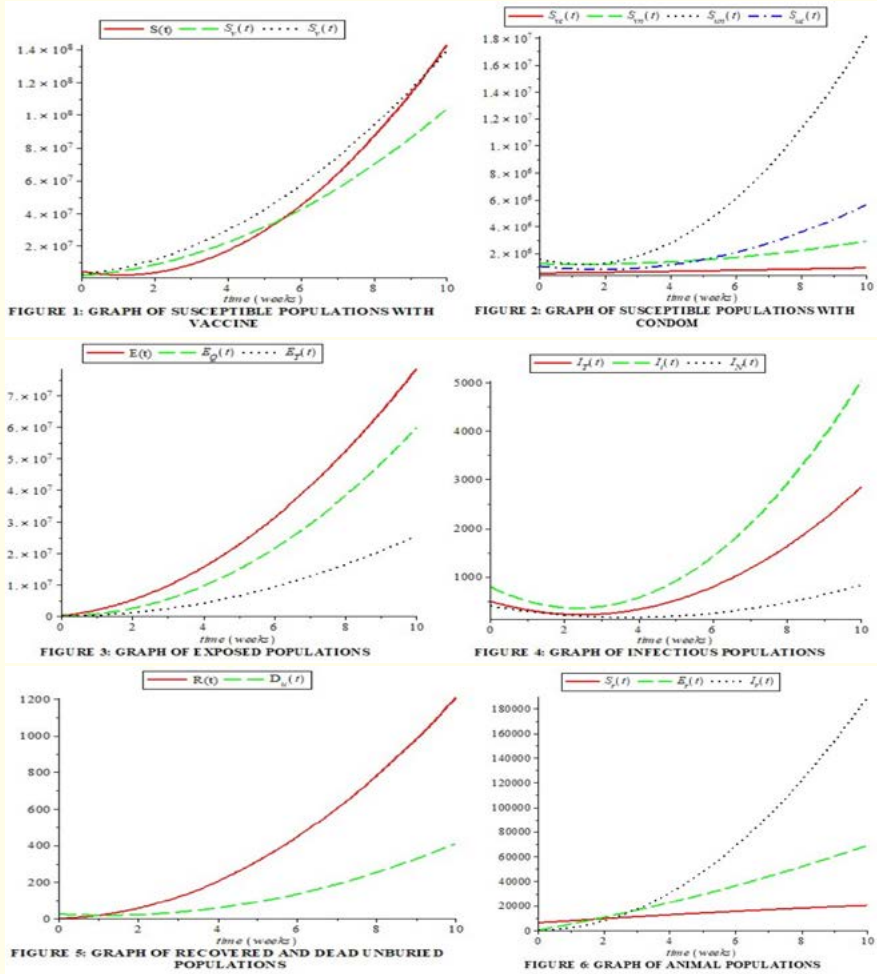

FIGURE 1: GRAPH OF SUSCEPTIBLE POPULATIONS WITH VACCINE

FIGURE 2: GRAPH OF SUSCEPTIBLE POPULATIONS WITH CONDOM

FIGURE 3: GRAPH OF EXPOSED POPULATIONS

FIGURE 4: GRAPH OF INFECTIOUS POPULATIONS

FIGURE 5: GRAPH OF RECOVERED AND DEAD UNBURIED POPULATIONS

FIGURE 6: GRAPH OF ANIMAL POPULATIONS

Figure 1 is the graph of LADM solution of $S(t), S_v(t)$ *and* $S_u(t)$, Figure 2 is the graph of LADM solution of $S_{vc}(t), S_{vn}(t), S_{uc}(t)$ *and* $S_{un}(t)$, Figure 3 is the graph of LADM solution of $E(t), E_Q(t)$ *and* $E_T(t)$, Figure 4 is the graph of LADM solution of $I_T(t), I_i(t)$ *and* $I_N(t)$, Figure 5 is the graph of LADM solution of $R(t)$ *and* $D_u(t)$ and Figure 6 is the graph of LADM solution of $S_r(t), E_r(t)$ *and* $I_r(t)$.

Figures 1- 6 shows that for each compartment of the model the LADM solution is approximate to the exact solution of the EVD model.

Conclusions

In this paper, we looked into a large eighteen compartmental fractional order EVD epidemic model with control measures; we applied the Laplace Adomian Decomposition Method (LADM) in solving the large fractional order EVD model and obtained the solution of zeroth, first and second order. The simulation of the LADM solution of each compartment gives a very good result which is approximate to the exact solution. We have been able to show that LADM can solve large compartmental fractional EVD disease model and obtain an approximate result. We therefore conclude that LADM is valid for solving a large number of parameters and equations models.

List of Abbreviations

EVD - Ebola Virus Disease.
LADM - Laplace Adomian Decomposition Method.
HPM - Homotopy Perturbation Method.
HAM - Homotopy Analysis Method.

Declarations

Availability of data and material

All data and material used for this work are available.

Competing Interests

There are no competing interests of any kind among the authors.

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Authors' Contributions

All authors contributed to the success of the work.

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