Medicon Engineering Themes
Volume 1 Issue 1 September 2021
Research Article

A State-Space Model from a Finite Element Method of Flexible chassis Model and Control It by (LQR) Problems

Mohamed Reyad Ghazy, Ayman M Abdel Haleem, Alla Ahmed Abdul-Rahman and Magdy Abdul-Aty Abdullah

Faculty of Engineering, Zagazig University, Zagazig, Egypt

***Corresponding Author:** Magdy Abdul-Aty Abdualla, Faculty of Engineering, Zagazig University, Zagazig, Egypt.

Received: September 04, 2021; **Published:** September 30, 2021

Abstract

The present paper aims to design an active vehicle system suspension by modification passive suspension into consideration the chassis is flexible to provide us the actual suspension image to make the best performance of the car and ride comfort. Creation of a State-Space half-car Model from a Finite Element, Model achieved by use MATLAB and Controlled by (LQR) Problems with its parameters Q and R which used in the determination of optimal car suspension systems design. The challenge is minimizing the chassis deflection; displacement and reduce the acceleration of suspension vibration. It is shown in our result that the model response to the Q and R-value is positive.

Keywords: Finite Element method; Beam equation; alf car vehicle; Simply supported beam

Introduction

The half-car model has a flexible chassis as the sprung mass which stands on two springs and two wheels act by two masses. The passive suspension parameters of a system are generally modified, which select for realizing certain equalization between road case and ride comfortably [1]. A half-car model analysis performed of linear and nonlinear semi-active dampers. Using Matlab-Simulink software, found the system response to a harmonic excitation of variable frequency and an impulse excitation [2]. By using a half-car model to discuss the car's dynamic response under random road excitations as input. The vehicle body mass, body mass moment of inertia, the front/rear wheels masses, damping coefficients, front/rear suspensions spring stiffness, the front/rear suspension locations distances to the center of gravity of the vehicle body, and front/rear tires stiffness are considered as random variables [3]. By studied using Matlab programming package, the control of linear half-car suspension by different control systems was developed to minimize the deflection which was born due to road disturbances. Comparisons between passive and active, linear simulation models have been carried out with different control schemes. The result of these comparisons was that the performance of the active linear model was attuned better than the passive linear model [4]. According to the ISO ride comfort techniques we can used the shock absorber with the auto-generation which permits collecting waste energy with an asymmetric ratio of compression [5] the rides vibration behavior Studied, the vehicle dynamic responses analyze, and the rider acceleration responses by using the FEM and its modal properties have been calculated. An explanation of the modal superposition theory is given and the possibility of incorporating this theory with Lagrange's equations for modeling a truck containing flexible subsystems is discussed and applied. Numerical results are presented for the truck, including power spectral densities and root mean square values of the vehicle dynamic response variables [6]. With a uniform load by using the finite element method, the vertical deflection of the chassis is considered. The governing differential equation is described by the Bernoulli beam which is a fourth-order differential equation [7] and the Lagrangian Method [8] many model reduction methods have been proposed in the reaches of approximating the high dimensional model with a lower order model. We envision that these systems-oriented model reduction methods complementing the existing Large-scale finite element models are routinely used in design and optimization for complex engineering systems [9]. The Models born from FEM are difficult to handle for control system design. so, the algorithm is presented that converts a system-which born from FEM into the state-space model of an interconnected system, thus decreases the complexity of synthesizing a distributed control strategy. So, we can say the FEM has an important role in the modeling of complex systems. A state-space representation of this System was obtained through FEM analysis which converts this model from lumped state-space form to interconnected form. then, it was designed by using the homotopy approach and decentralized controller [10]. He state-space model was used as reference one for modal decreasing whereby a balanced reduction method was applied. The reliability of the model is verified by comparative between the impulse response of the full model and the miniature model in the time domain. this result was Obtained afterward to set model dynamic behavior in the state space by using MATLAB software [11]. due to irregular road surface, vibration on the passenger seat generated so the active suspension system designed to reduce it [12]. Or low-cost semi-active suspension replacing by passive suspension [13]. In the present paper, The Galerkin method of finite element [14], by substituting a test function to the governing equations. We will explain the use of the Galerkin Finite Element Method to solve the beam equation with the assist of Matlab (fig (1)). Complex partial differential equations that describe this system can be decreased to a set of linear equations that can easily be solved by this method [14]. Also, we can adjust the system using the state-space equation by using the linear quadratic regulator to control the vibration amplitude and its acceleration.

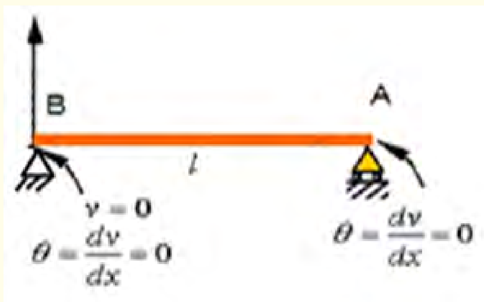

Figure 1: Galerkin Finite Element.

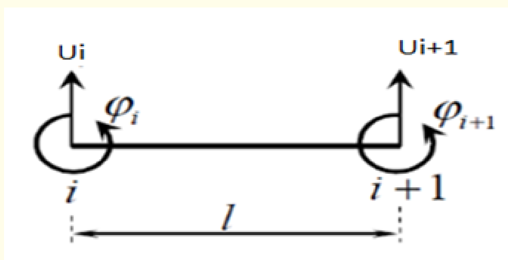

Figure 2: Beam Element.

Governing Equation

The Euler-Bernoulli beam theory as shown in figure (2), the transverse deflection u(x) of the beam is governed by the fourth-order differential equation [15].

$$\frac{d^2}{dx^2}\left[y(x)\frac{d^2u}{dx^2}\right] = f(x,u), \qquad (1)$$

0 ≤ x ≤ L

Subject to the free end boundary conditions

$$u(0) = a_0, \quad \frac{d^2 u(0)}{dx^2} = b_0, \quad u(L) = a_L, \quad \frac{d^2 u(L)}{dx^2} = b_L \,. \qquad (2)$$

Assuming the function y(x) = I $*E$ is constant, where (I and E) is the moment of inertia and modulus of elasticity of the beam respectively. The transversely distributed load is $f\ (x, u)$, equal to$q$ (x) u(x) + $p\ (x)$, in the linear case, where $u(x)$ is the deflection of the beam, $q\ (x)$ is the coefficient of ground elasticity and $p(x)$ is the uniform load applied to the beam. by substituting from the value of equation (1) we get the following.

$$\frac{d^2}{dx^2}\left[EI\frac{d^2 u}{dx^2}\right] = q(x)u(x) + p(x), \quad 0 \le x \le L, \qquad (3)$$

When the beam is fixed at ends and u (0) = 0, the solution of $u\ (x)$ describes the deflection of the beam under the load $p\ (x)$. In this case, the governing equations become as (4).

$$\frac{d^2}{dx^2}\left[EI\frac{d^2 u}{dx^2}\right] = p(x), \ \ 0 \le x \le L \qquad (4)$$

For simple data functions $f\ (x, u)$ and y(x), (in equation (1)) the exact solution of beam equation with boundary condition can be found by standard methods that are well known in the literature of ordinary differential equations and their applications. For more developed data functions, when exact methods fail, numerical methods can be successfully applied to find an approximate solution for a broad class of boundary value problems. In the next section, we shall utilize the Galerkin Finite Element Method (FEM) to solve equation (4) [17].

Galerkin Finite Element Method

Stage one of the Galerkin FEM is the determination of the domain [15]. which is divided into a finite set of line elements, each of which has two nodes. Stage two of the Galerkin method is to obtain the weak form of the differential equation. For this purpose, we multiply the residual of a differential equation (4) by a weight function w(x) and integrate it by parts to evenly distribute the order of differentiation on u and w. The result is the equation [18].

$$\int_0^L \left[EI\frac{d^4 u}{dx^4} - p(x)\right] w\, dx = \left[EI\frac{d^3 u}{dx^3} w\right]_0^L + \left[EI\frac{d^2 u}{dx^2}\frac{dw}{dx}\right]_0^L$$
$$+ \int_0^L \left[EI\ \frac{d^2 w}{dx^2}\frac{d^2 u}{dx^2} - pw\ \right] dx = 0 \quad (5)$$

by balance the weight function to the approximating function wi = Ni,

we get

$$\int_0^L \left[EI\frac{d^4 u}{dx^4} - p(x)\right] w\, dx = [EIu_{xxx}N_i]_0^L - \left[EIu_{xx}N_{i,x}\right]_0^L + \int_0^L EIN_{i,xx}u_{xx}\, dx\_ \int_0^L pN_i dx = 0 \quad (6)$$

After obtaining the weak form, we proceed to choose the suitable elements approximating functions. It can be noted that the highest order of the derivative on (x) in the weak form (5, 6) is three; therefore, we choose an approximating function that is thrice differentiable. this request is satisfied by cubic interpolation polynomial, where these cubic interpolation functions are known as Hermit cubic interpolation functions [18].

$$N_1 = \frac{1}{L^3}(2x^3 - 3x^2L + L^3),$$
$$N_2 = \frac{1}{L^3}(x^3L - 2x^2L^2 + xL^3), \quad N_3 = \frac{1}{L^3}(-2x^3 - 3x^2L)$$
$$N_4 = \frac{1}{L^3}(x^3L - x^2L^2) \quad (7)$$

Where N_1, N_2, N_3, & N_4 are called the shape functions for a beam element. For the beam element, N_1= 1 when evaluated at node 1 and N_1=0 when evaluated at node 2. And from equation (6) the stiffness matrix and force vector are given as follow

$K_{ij} = EI\int_0^L \frac{d^2N_i}{dx^2}\frac{d^2N_j}{dx^2}\,dx$, And the force vector is $f_i = \int_0^L pN_i\,dx$

For the first element

$$K_{11} = \int_0^L \frac{d^2N_1}{dx^2}\frac{d^2N_1}{dx^2}\,dx = \int_0^L \frac{1}{L^3}(12\,x - 6L)\frac{1}{L^3}(12x - 6L)dx = \frac{1}{L^6}\int_0^L 144x^2 - 144\,xL + 36\,L^2dx =$$
$$\frac{1}{L^6}\int_0^L[48\,x^3 - 72xL + 36xL]_0^L = \frac{1}{L^6}[12L^3] = \frac{12}{L^3} \quad (8)$$

The remaining elements are found similarly. The stiffness matrix [18] becomes

$$k_{i,j} = \frac{EI}{L^3}\begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix} \quad (9)$$

Similarly, we can obtain the force vector matrix. The first value in the force vector is evaluated below [17].

$$\int_0^L p\left(1 - \frac{3x^2}{h^2} + \frac{2x^3}{h^3}\right)dx = p[x - \frac{x^3}{h^2} + \frac{x^4}{2h^3}]_0^L = p\left(L - \frac{L^3}{L^2} + \frac{L^4}{2L^3}\right) = p(\frac{L}{2}) \quad (10)$$

The remaining values are obtained in a similar manner using their corresponding shape functions [18]. The resulting force vector is given as

$$f = \frac{Lp}{2}\begin{bmatrix} 1 \\ 6L \\ 1 \\ -6L \end{bmatrix} \quad (11)$$

The corresponding system from equation (9), (11) can be represented as

$$f = \frac{Lp}{2}\begin{bmatrix} 1 \\ 6L \\ 1 \\ -6L \end{bmatrix} = \frac{EI}{L^3}\begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix}\begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix} \quad (12)$$

Force = [Stiffness] *[Displacement].

The equations system was solved by using MATLAB software. The element displacements were solved under different conditions prescribed [18].

Derivation of the stiffness matrix for a spring element

The local nodal force at node 1 for the spring element is associated with the local axis y. the local axis acts in the direction of the spring to be able to directly measure displacement and force along with the spring. The local nodal displacements are $\tilde{u}_1, \tilde{u}_2, \tilde{u}_3, \tilde{u}_4$ for the nodal of the elements as shown in Fig. (3). the force at node one can be written as follow:

For node number two as follow.

$$\tilde{f}_{y1} = (\tilde{u}_1 - \tilde{u}_2)k \qquad (13)$$

For node number two as follow.

$$\tilde{f}_{y2} = (\tilde{u}_2 - \tilde{u}_1)k \qquad (14)$$

Or we can write the equation 13, 14 as follow.

$$\begin{Bmatrix} \tilde{f}_{y1} \\ \tilde{f}_{y2} \end{Bmatrix} = \begin{bmatrix} k & -k \\ -k & k \end{bmatrix} \begin{Bmatrix} \tilde{u}_1 \\ \tilde{u}_2 \end{Bmatrix} \qquad (15)$$

So, we can write the stiffness matrix for element spring assuming k1 equal k2

$$k_1^e = k_2^e = k \qquad (16)$$

Finite Element modelling

By using FEM to calculate the node beam displacement of the and its associated modes of the flexible beam supported by two springs at its ends (without damping for our case). The stiffness and mass element matrices that derived for the beam bending motion assuming Euler_ Bernoulli bending theory. the beam is discretized (N-1) two-node elements generating N nodes. Fig. (2) shows an element that has two degrees of freedom per node, u_i, φ_i represents displacement and deflection of $(i)^{th}$ node, u_{i+1} and φ_{i+1} represents displacement and deflection of $(i+1)^{th}$ node respectively.

The global stiffness matrix for our case

From equation (12), (16) for the beam element and spring element respectively (her for the study assume the beam is one element) we can get the global stiffness matrix for all the system as follow:

$$K_G = \frac{EI}{L^3} \begin{bmatrix} \frac{kL^3}{EI} & -\frac{KL^3}{EI} & 0 & 0 & 0 & 0 \\ -\frac{KL^3}{EI} & 12+\frac{kL^3}{EI} & 6L & -12 & 6L & 0 \\ 0 & 6L & 4L^2 & -6L & 2L^2 & 0 \\ 0 & -12 & -6L & 12+\frac{kL^3}{EI} & -6L & -\frac{KL^3}{EI} \\ 0 & 6L & 2L^2 & -6L & 4L^2 & 0 \\ 0 & 0 & 0 & -\frac{KL^3}{EI} & 0 & \frac{kL^3}{EI} \end{bmatrix} \qquad (17)$$

By using the boundary condition for the system. Then a global stiffness matrix becomes as follow

$$K_G = \frac{EI}{L^3} \begin{bmatrix} 12+\frac{kL^3}{EI} & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12+\frac{kL^3}{EI} & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix} \qquad (18)$$

We must know the global stiffness for our modeling will take another form because the beam divides into two elements and assuming the boundary condition for the propped cantilever beam Fig. (1).

Drive the element mass matrix for the beam

$$M = \int_0^L N^T m N \, dx$$

Where: $m=\rho AL$, N= [N_1 N_2 N_3 N_4]we can get the value of (N) from equation (7). Now, we can get the value of ($N^T N$) as follow

$$N^T N = \begin{bmatrix} N_1 \\ N_2 \\ N_3 \\ N_4 \end{bmatrix} [N_1 \quad N_2 \quad N_3 \quad N_4]$$

Or

$$N^T N = \begin{bmatrix} N_1N_1 & N_1N_2 & N_1N_3 & N_1N_4 \\ N_2N_1 & N_2N_2 & N_2N_3 & N_2N_4 \\ N_3N_1 & N_3N_2 & N_3N_3 & N_3N_4 \\ N_4N_1 & N_4N_2 & N_4N_3 & N_4N_4 \end{bmatrix} \quad (19)$$

$$N_1N_1 = \left\{\frac{1}{L^3}(2x^3 - 3x^2L + L^3)\right\}^2 = \frac{1}{L^6}(2x^3 - 3x^2L + L^3)^2 \ldots \ldots \ldots$$
$$= 4x^6 - 12x^5L + 9x^4L^2 + 4x^3L^3 - 6x^2L^4 + L^6$$

Now,

$$\frac{1}{L^6}\int_0^L N_1N_1 dx = \frac{4}{7}L^7 - 2L^7 + \frac{9}{5}L^7 + L^7 - 2L^7 + L^7 = \frac{L^7}{L^6}\left(\frac{13}{35}\right) = \frac{L * 156}{420}$$

For the element N_1 N_2, and we can do all elements of equation (19) in the same way to get the global mass matrix (20) for beam in our system as follow:

$$M = \frac{\rho AL}{420}\begin{bmatrix} 156 & 22L & 54 & -13L \\ 22L & 4L^2 & -13L & -3L^2 \\ 54 & -13L & 156 & -22L \\ -13L & -3L^2 & -22L & 4L^2 \end{bmatrix} \quad (20)$$

System input

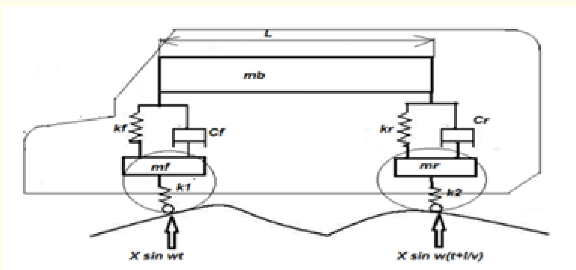

Figure 3: Sketch half-car Model System.

Fig. 3 shows the sketch half-car model system. The suspension model consists of a chassis as a beam and two axles. Linear spring supports the chassis without includes the tires. The model excited through the tires as a base motion as the input displacement from the road equals sinusoidal wave at the front tire (x= X sin wt.) and the same value at the rear tire additional time delay (x=X sin wt. +L/v) as shown in figure 4. Where X is the amplitude of displacement and equal 0.1, L the length of the chassis, and v is the velocity of the vehicle [1]. the tire contacts act as the constraints of this system. Thus, consider tires are static. so, we can replace it with two pins. Where cf, $cr$is the dampers of front and rear respectively. and (kf and kr) is the spring for the front and rear suspension respectively. (k1 and k2) is the rear and front stiffness tire respectively. Mb is the sprung mass, (mf and mr) is the front and rear tire respectively. L is the length of the chassis. Using the above notations, the mathematical relations are derived as follows (see appendix A, B, and C)

State-space solutions

The generalized state-space representation as follow:

$$x' = Ax + B*F = \begin{bmatrix} 0 & 1 \\ -M^{-1}*K & -M^{-1}*C \end{bmatrix} * x + \begin{bmatrix} 0 \\ -M*F \end{bmatrix} * F \qquad (21)$$

$$y=Cx+Du=[0 \quad 1]*x+[0]*u (22)$$

Where M, C, and K are the full mass, damping, and stiffness matrices respectively, x is the vector of states, F is the load vector, y is the output vector, and [B] and [C] are rectangular matrices with ones on the desired DoFs positions and zeros everywhere else in control engineering, the state space representation is a mathematical model of a physical system as a set of input, output and state variables related by first-order differential equations. The state-space means then-dimensional space whose coordinate axes consist of a lot of state variables axis. A point in the state space represented by axis. The state space is converting the second-order differential equations for dynamic system to the first-order state space dynamic system which has a certain advantage over the second-order form descriptions. A seconder equation can be transformed into a first adequation and vice versa. The State-Space block implements a system whose behavior is defined as the following where (n) the number of Dof, (m) the number of inputs, and (r) the number of outputs as shown in figure (4).

	n	m
n	**A**	**B**
r	C	D

Figure 4: The State-Space Block Matrix Dimension.

MATLAB Solution to LQR problems

Control systems are designed for the spacecraft [16], by using quadratic performance indexes with MATLAB. the minimization of quadratic performance indexes which are used for the system design is based on MATLAB has a command that gives the solution to the associated algebraic Riccati equation (ARE) and determines the optimal control gain matrix [16].

"LQR" Command for Solving ARE (Appendix A)

The continuous-time, linear, quadratic regular problem and the associated Riccati equation are solved bythe MATLAB command " LQR (A, B, Q, R)". The previous command calculates the optimal feedback gain matrix "K" such that the feedback control law and minimizes the performance subject to the constraint equation [16] as following.

[P, S, K] = lqr (A, B, Q, R) = lqr (A, B, C'* C, R)

Where (Appendix A):

P = Positive definite symmetric matrix
S = Poles of the system
K = Optimal control gain

Where:
Q= diagonal one matrix has dimension equal number degree of freedom mutably two (Appendix A)

Results

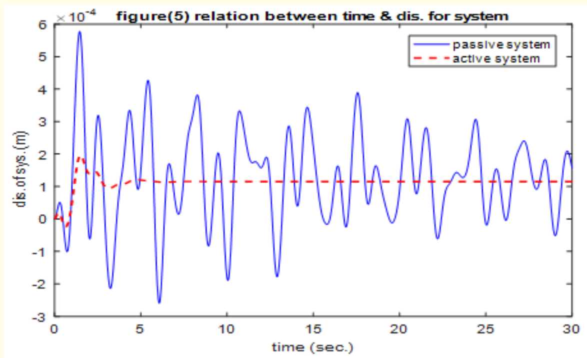

Figure 5: Relation Between time & displacement for system.

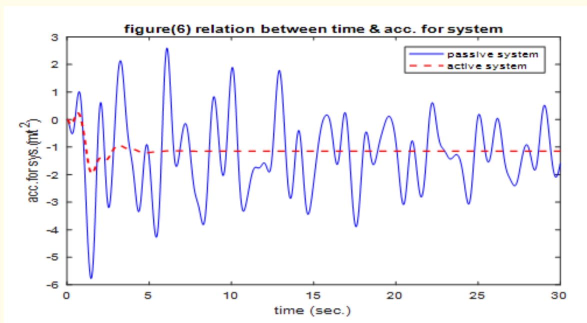

Figure 6: Relation between time & acceleration for system.

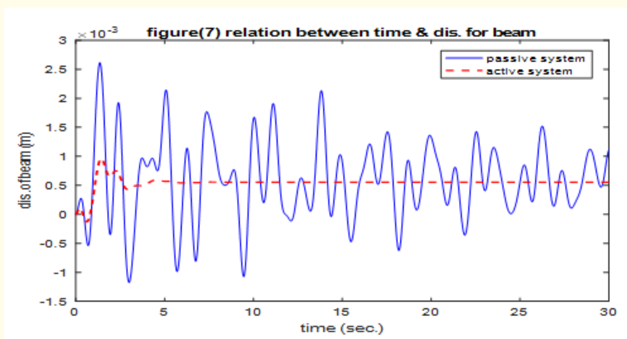

Figure 7: Relation between time & displacement for beam.

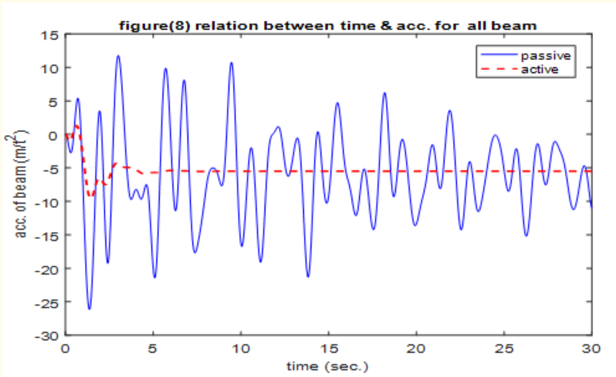

Figure 8: Relation between time & acceleration for beam.

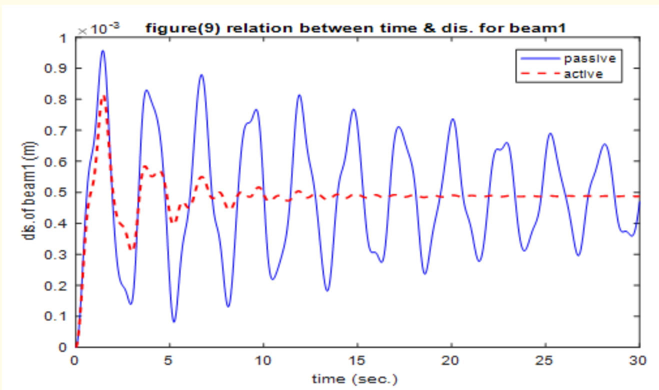

Figure 9: Relation between time & displacement for beam1.

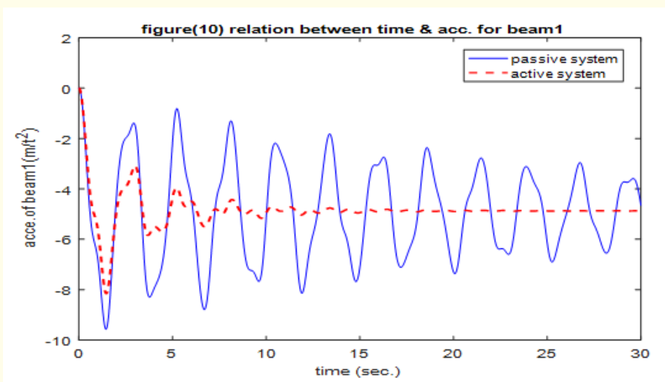

Figure 10: Relation between time &acceleration for beam 1.

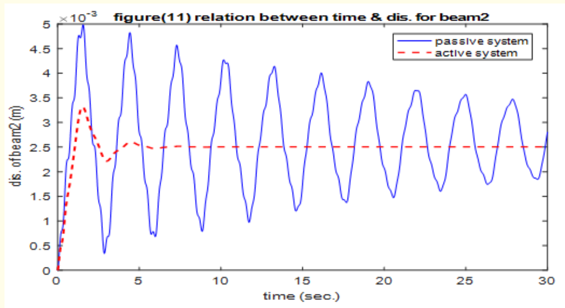

Figure 11: Relation between time & displacement for beam 2.

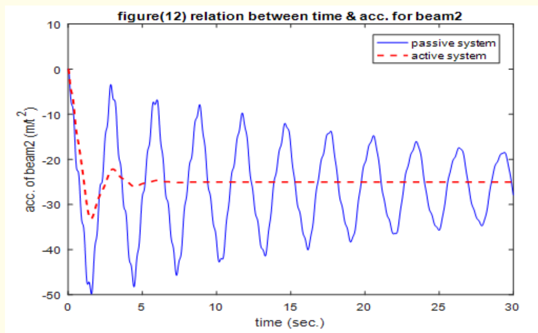

Figure 12: Relation between time &acceleration for beam 2.

Discussion

We get the mass matrix and stiffness matrix by using the finite element method and using state space to get a dynamic response. By finite element method theory, we can say the error reduces, as increasing the number of elements improves the accuracy of the solution. the figures (5-12) show the results obtained for homogenous boundary conditions for the system at all and chassis only. It appears from the figures the passive system (continuous line) greater than the active system (dashed line) by three times. The transferring from a passive to an active system is satisfied by the userthe value of Q & R Parameter Using Matlab code to do the program (Appendix A). the finite element method and state-space which can be aid the designer to make the system activating assuming the beam are elastic by minimized the amplitude of fluctuation and its time by changing the parameter value of Q & R. Fig. (5) and fig. (6) shows the relationship between time, displacement, and time, acceleration for the system. Note the amplitude of displacement and acceleration reduces depending on the control of the system. Fig. (7) and fig. (8) show the relationship between time, displacement and time, acceleration for the beam (chassis) only. the amplitude of displacement and acceleration reduces depending on the control of the system. Fig. (9) and fig. (10) show the relationship between time, displacement, and time, acceleration for the first element of the beam. Note the amplitude of displacement and acceleration reduces depending on the control of the system. Fig. (11) and fig. (12) show the relationship between time, displacement and time, acceleration for the second element of the beam. Note the amplitude of displacement and acceleration reduces depending on the control of the system.

Conclusions

The present paper objective analysis the suspension system behavior for a half-car model which influenced by road irregular. The displacement of the half-vehicle with tire via Matlab depended upon the finite element method and state-space control. Considered under load at the contact point between the tire and the road. Get the mass matrix and stiffness matrix for chassis by finite element method. Graphs are presented and discussed the displacement and acceleration at passive and active cases for a system at all and chassis as two elements. Note the system under control and we can reach to our aim of car control and comfort of the ride.

References

1. Anupam M., et al. "Comparative Analysis of an Elastic 2 Degree of Freedom Half Car Model with a Rigid two Degree of freedom Model". SAE International (2012).
2. Urszula F and Jan L. "vibration analysis of the half-car model with sime-active damping". Journal of Theoretical and Applied Mechanics 54.2 (2016): 321-332.
3. W Gao., et al. "A half-car model for dynamic analysis of vehicles with random parameters". 5th Australasian Congress on Applied Mechanics, Brisbane, Australia (2007).
4. Ammar MH., et al. "Modeling, Simulation, and Control of Half Car Suspension System Using Matlab/Simulink". International Journal of Science and Research (IJSR) 7.1 (2016): 79.57.
5. Anas A Samn., et al. Zagazig University, Egypt "Study of a Mono-Tube Hydraulic Energy Harvesting Shock Absorber" SAE International Journal of Passenger Cars - Mechanical Systems 12.3 (2019):169-182.
6. Ibrahim IM., et al. "Effect of Frame Flexibility on the Ride Vibration of Trucks". Computers and Structures 58.4 (1996): 709-713.
7. EP Popov., et al. Euler-Bernoulli Beam Equation, Mechanics of Materials, Prentice-Hall, K.S (1978).
8. David Morin. "The Lagrangian Method" (2007).
9. William G., wt al. "Order Reduction for Large Scale Finite Element Models: a Systems Perspective" 3.3 (2005): 337-362.
10. Syed FR., et al. "A Tool for Converting FEM Models into Representations Suitable for Control Synthesis". The International Federation of Automatic Control Seoul, Korea (2008).
11. Aleksandar K., et al. "State Space Modeling From Fem Model Using Balanced Reduction". International Conference Industrial Engineering and Environmental Protection (IIZS 2015) Zrenjanin, Serbia (2015).
12. Y Susatio., et al. "Design of half-car active suspension system for passenger riding comfort" Series: Journal of Physics: Conf. Series 1075.1 (2018).
13. Mostafa Ghoniem A., et al. "Control of a new low-cost semi-active vehicle suspension system using artificial neural networks". Faculty of Engineering, Alexandria University 59.5 (2020): 4013-4025.
14. S djerid M Baccouch. "Galerkin method" Scholarpedia 5.10 (2010): 10056.
15. JAM Crrer., et al. "Analysis of Euler-Bernoulli and Timoshenko beams by the boundary element method" 2012 Conference: 10th World Congress on Computational Mechanics At: São Paulo, Brazil.
16. AA Afis and SA Arisekola. "The Use of Matlab in the Solution of Linear Quadratic Regulator (Lqr) Problems". Computing, Information Systems, Development Informatics & Allied Research Journal 5.4 (2014).
17. Cheng Yung Ming. "Solution of Differential Equations with Applications to Engineering Problems". Dynamical Systems - Analytical and Computational Techniques, Mahmut Reyhanoglu, Intech Open (2017).
18. RL Taylor and JZ Zhu. "The Finite Element Method: Its Basis and Fundamentals" First published in 1967 by McGraw-HillFifth edition published by Butterworth-Heinemann 2000 Reprinted 2002 Sixth edition 2005.

Appendix A

```
bB=alpha*massB+beta*stiffB; %   -----damping matrix ----
IB=eye(GDof-2);
AB11=zeros(GDof-2);
AB12=IB;
MASSB=inv(massB);
AB21=-MASSB*stiffB;
AB22=-MASSB*bB;
BB11=zeros(GDof-2,1);
```

```
BB21=MASSB*forceB;
CB11=eye(1,GDof-2);
CB12=zeros(1,GDof-2);
AB=[AB11 AB12;AB21 AB22];
BB=[BB11;BB21];
CB=[CB11 CB12];
D=[0];
GB = ss(AB,BB,CB,D);
cGB = canon(GB);
Q=eye(GDof+2);
R=[1];
[k2,s2,e] = lqr(AB,BB,Q,R);
GBk2 = ss(AB-BB*k2,BB,CB,D);
cGBk2 = canon(GBk2);
t = [0:0.01:30];
u=0.01*ones(size(t));
% u = 0.1*sin(pi*t);
figure;
[Y2,X]=lsim(cGB,u,t);
[Y3,X]=lsim(cGBk2,u,t);
h=plot(t,Y2,'b',t,Y3,'r--');
set(h(1),'linewidth',0.8);
set(h(2),'linewidth',1.5);
xlabel('time (sec.)');
ylabel('dis.of beam (m)');
legend ('passive system','active system');
title ('figure(7) relation between time & dis. for beam');
n=3001;
for i=2:n
Y2dot(i)=Y2(i)-Y2(i-1)/t(i)-t(i-1);
Y2dotdot(i)=Y2(i)-Y2(i-1)/(t(i)-t(i-1))^2;
Y3dot(i)=Y3(i)-Y3(i-1)/t(i)-t(i-1);
Y3dotdot(i)=Y3(i)-Y3(i-1)/(t(i)-t(i-1))^2;
end
vel2=Y2dot;
vel3=Y3dot;
acc2= Y2dotdot;
acc3= Y3dotdot;
%   figure (5);
%   h=plot(t,vel2,'b',t,vel3,'r--');
%  set(h(1),'linewidth',0.8);
%   set(h(2),'linewidth',1.5);
%  xlabel('time (t)');
%   ylabel('vel. of beam (m/t)');
```

```
figure;
h=plot(t,acc2,'b',t,acc3,'r--');
set(h(1),'linewidth',0.8);
set(h(2),'linewidth',1.5);
xlabel('time (sec.)');
ylabel('acc. of beam (m/t^2)');
legend ('passive ','active');
title ('figure(8) relation between time & acc. for  all beam');
```

Volume 1 Issue 1 September 2021