Medicon Engineering Themes
Volume 3 Issue 6 December 2022
Article Type: Review Article
ISSN: 2834-7218

Complex Solution of Engineering Problems by Graphic Methods

Aleksandr Yurievich Brailov*

Odessa Professional College of Computer Technologies, Odessa State Ecological University, Ukraine

***Corresponding Author:** Aleksandr Yurievich Brailov, Odessa Professional College of Computer Technologies, Odessa State Ecological University, Ukraine.

Received: October 22, 2022; **Published:** November 01, 2022

Abstract

A geometrical problem is to determine the location of the image equidistant from four points that do not coincide in three-dimensional space. A practical example of this problem is to determine the location of the lamp equidistant from four lamps, arbitrarily located in the hall of an art gallery, a sports hall, on a site in a park and other objects. A method has been developed for the graphical determination of the geometrical place of an image equidistant from four points that do not coinciding in three-dimensional space. A methodology for the graphical solution of engineering geometric problems is proposed.

Keywords: Geometric Image; Engineering Problem; Graphic Solution; Point; Sphere; Solution Methodology

Stereotyped approach to problem solving

When creating modern architectural structures and engineering structures, one of the challenging problems is to determine the geometrical place of the image, equidistant from four points that do not coincide in the three-dimensional space.

A stereotype formulation of the problem is: "Construct the *locus of points* equidistant from points *A, B, C, D*" [1-3]. As an example, a complex drawing of four non-coincident points is given in Figure 1.

The *initial assumption* that the desired locus of points is the axis of a cylinder, on the surface of which points *A, B, C, D* are located, may leads to the result shown in Figure 2.

According to Figure 2, the solution of the problem starts from the straight line segment defined by the points *A* and *D*, which is a generatrix line of a circular cylinder. The constructed *i*-axis is parallel to the generatrix line *AD*. The i_5 is a projection of the axis *i* equidistant from the *projections* A_5, D_5, B_5, C_5 of the points *A, B, C, D*, respectively.

For the method described above to solve the problem, the obtained result is not unique.

The number of possible results obtained using the described method is the number of unordered subsets *{A, B}, {A, C}, {A, D}, {B, C}, {B, D}, {C, D}* of two elements (r = 2) of the set *{A, B, C, D}* containing four elements (k = 4).

The number of all possible results C_k^r is equal to the number of combinations of two (r = 2) different points of the set containing four (k = 4) points:

$$C_k^r = k!/[r!\cdot(k-r)!]=4!/[2!(4-2)!]=4!/(2!\cdot 2!)=24/4=6.$$

Thus, the number of all possible results for the described method for solving the problem is six.

Using the graphical method of changing the projection planes [1-5], it is easy to demonstrate that all six results of the presented solution are different.

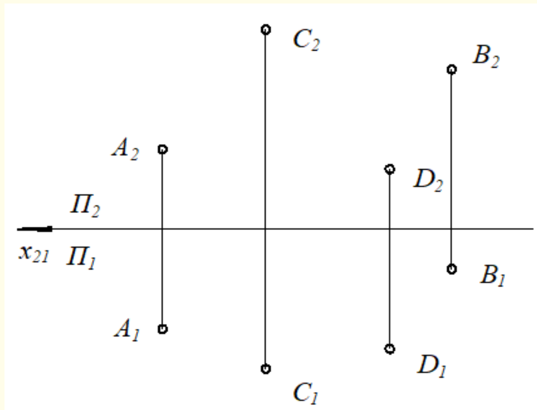

Figure 1: Complex Drawings of Mismatched Points.

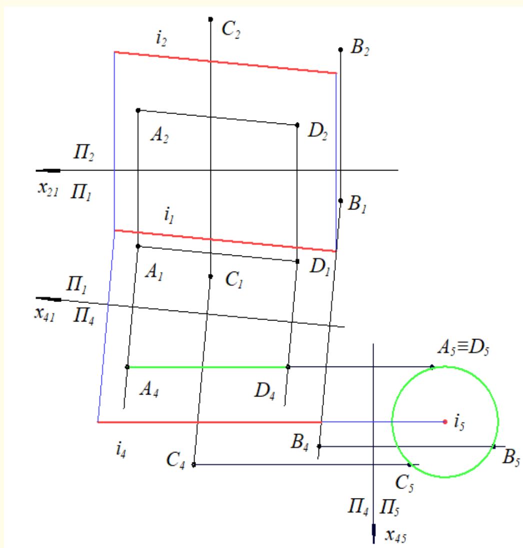

Figure 2: Solution of the problem for the assumption that the desired locus of points is the *i*-axis of the cylinder.

Let us show that the stereotype approach to constructing the locus of points equidistant from points *A, B, C, D*, which are arbitrarily located in the three-dimensional orthogonal space and do not coincide with each other, does not give the true result (Fig. 3).

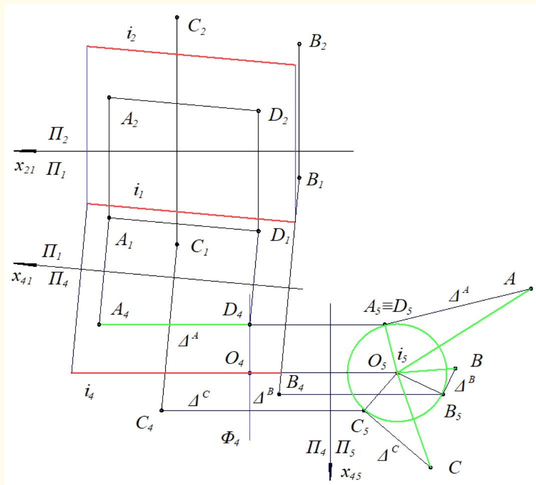

Figure 3: The natural values of the distances between an arbitrary point *O* of the *i*-axis and points *A, B, C, D* are different.

The plane Φ intersecting the cylinder is constructed perpendicular to the plane of projections Π_4. The plane Φ passes through point *D* perpendicular to the *i*-axis of the cylindrical surface, and point *O* is the intersection between the plane Φ and the *i*-axis of the cylinder.

All four points *A, B, C, D* belong to a cylindrical surface.

Since the plane Φ is perpendicular to the *i*-axis of the cylinder and is parallel to the plane of projections Π_5, the distance between points *O* and point Din the plane of projections Π_5 projects in its natural size.

To test the validity of the assumption that "the desired locus of points is the axis of the cylinder, on the surface of which points *A, B, C, D* are located", it remains to determine the natural values of the distances between an arbitrary point *O* of the *i*-axis and points *A, B, C*.

The distances between an arbitrary point *O* and points *A, B, C* are determined by the method of a right-angled triangle [1-5].

Graphical constructions shown in Fig. 3 present the visual proof that, on the "constructed geometrical place of points", the distances from an arbitrarily chosen point *O* to points *A, B, C, D* are different.

Consequently, the constructed locus of points *i* is not the true solution of the problem.

A straight line would be indeed the locus of points equidistant from points *A, B, C, D* if and only if all the original points *A, B, C, D* are located on the same circle of the cylinder. In this case, the resulting straight line must pass through the center of the original circle and be perpendicular to the plane containing this circle. Any circle on the surface of a circular cylinder is a guide line and all its points are equidistant from an arbitrary point on its axis.

The validation test of any of the six possible results (Fig. 3) confirms the need to develop a method for obtaining a (true) real result through a correctly formulation of the problem.

Correct approach to solving the problem

The development of the required solution begins with the correct formulation of the problem and an assumption about the nature of the resulting geometric image. If the assumption is incorrect, it is impossible to obtain the correct result.

It makes sense to generalize the formulation of the problem being solved as follows: Determine the locus of the *image* equidistant from four points *A, B, C, D*, which are not coincident, in the three-dimensional space. Provided that any combination of three points out of the given four points does not belong to a straight line and a plane of level [3-5].

Then the *initial* assumption that the desired locus of points is the axis of the cylinder, on the surface of which points *A, B, C, D* are located, will take on a different form.

Assumption 1. The locus of the desired *image* equidistant from four non-coincident points *A, B, C, D* in the three-dimensional space is a *point*, denominated herein as *K*, but not a straight line (cylinder axis).

Point *K* is equidistant from the given points *A, B, C, D* if it is the center of the spherical surface on which points *A, B, C, D* are located.

To construct a complex drawing of the *desired image*, it is necessary to graphically determine the position of the *center K* and the actual value of the *radius* R_{HB} of the spherical surface containing points *A, B, C*, and *D*.

The solution of such a sub problem is related to the assumption of a possible way of constructing the center*K* of the spherical surface containing points *A, B, C, D*.

Assumption 2. The center *K* of the spherical surface with points *A, B, C, D* is located at the intersection of the *mid-perpendiculars* restored to all four flat faces of the tetrahedron, the vertices of which are points *A, B, C, D* that belong to the spherical surface.

Each such perpendicular to the flat face of the tetrahedron *ABCD* is the locus of points equidistant from the three vertices of the corresponding face.

The intersection of *two* mid-perpendiculars of any two flat faces of the tetrahedron inscribed in the sphere is the point *K* equidistant in three-dimensional space from the given four non-coincident points *A, B, C, D*.

If the formulated assumptions 1 and 2 are valid, then three subproblems are solved using the graphic methods of engineering geometry [1-5] as:

1. The construction of complex drawings of the four *centers* of the circles circum-scribed around each triangular face of the tetrahedron.
2. The construction of complex drawings of the *midpoint perpendiculars* reconstructed through the centers of the circumscribed circles to the flat faces of the tetrahedron *ABCD*.
3. The construction of a complex drawing of the *point of intersection* of the mid-perpendiculars.

The construction of a complex drawing of the center of a circle described through the vertices of the triangular face of the tetrahedron is performed by changing the projection planes using the main lines of the flat face [1-5].

The result is O^A (O_1^A, O_2^A) of the graphical solution of such a subproblem, for ex-ample, for the face *BCD* is shown in Fig. 4.

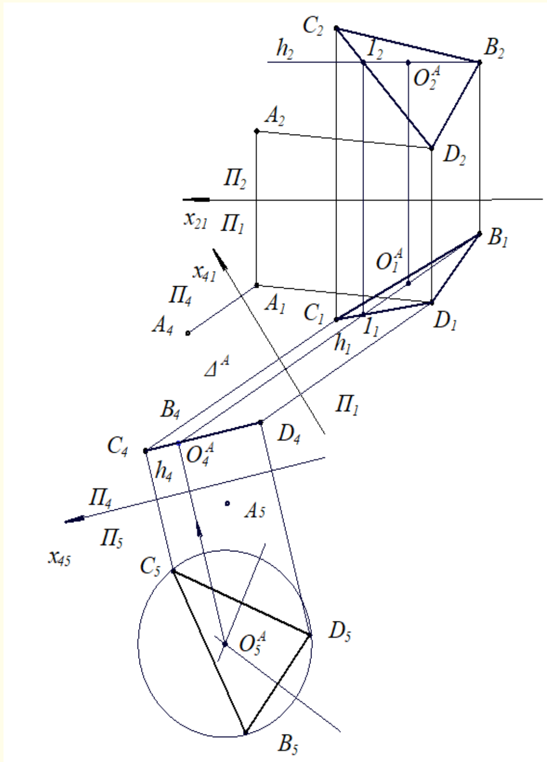

Figure 4: Complex drawing of the center O^A (O_1^A, O_2^A) of the circle de-scribed through the vertices of the triangular face *BCD* of the tetrahedron.

The *algorithm* for constructing a complex drawing of the center of a circle de-scribed through the vertices of the spatial triangular face of the tetrahedron consists of seven steps. Focusing on the face *BCD*, we have:

1. A complex drawing h (h_1, h_2) of the horizontal line belonging to the face *BCD* is constructed.
2. Perpendicular to the horizontal projection plane Π_1, a new projection plane Π_4 is introduced. Then, the x_{14} axis is drawn perpendicular to the horizontal projection h_1 of the horizontal h.
3. New lines of projection links are constructed from the horizontal projections A_1, B_1, C_1, D_1 of the tetrahedron vertices perpendicular to the new x_{14} axis.
4. On the lines of projection links in the plane of projections Π_4, the distances equal in magnitude to the distances from the x_{21} axis between the planes of the replaced system Π_2/Π_1 to the corresponding frontal projections A_2, B_2, C_2, D_2 of the tetrahedron vertices are indicated by serifs marks.
   Graphically, these different distances are marked with dashes or any other graphical signs.
5. At the intersection of the lines of projection links and serifs for the correspond-in g distances, new projections A_4, B_4, C_4, D_4 *of* the tetrahedron vertices are graphically indicated and highlighted.
   Since the horizontal h of the plane of the *BCD* face is perpendicular to the plane of projections Π_4, the projection $B_4C_4D_4$ of this face is a collective segment of a straight line.
   Now if the projection plane Π_5 orthogonal to the plane Π_4 and parallel to the collective projection $B_4C_4D_4$ of the flat face *BCD* is introduced, then the triangular face of *BCD* is projected onto this new plane Π_5 in true size.

6. Similarly, repeating steps 2-5 of the developed algorithm, projection $B_5C_5D_5$ is constructed on the new projection plane Π_5. This projection is equal to the true value of the triangular face *BCD* of the tetrahedron *ABCD*. The center O^A of the circum-scribed circle is located at the intersection of the perpendiculars to the sides B_5C_5, C_5D_5, B_5D_5 of the triangle $B_5C_5D_5$.
7. Following the rules of the method of changing projection planes [1-3], projections O_4^A, O_1^A, O_2^A of the center O^A of the circum-scribed circle for the triangular face *BCD* of the tetrahedron *ABCD* are determined using reverse projections.

Complex drawings of the centers of O^B, O^C, O^D of the circumscribed circles other faces *ACD, ABD, ABC* are constructed in the same way (Fig. 5).

The construction of a complex drawing of the *middle perpendicular*, reconstructed through the center of the circle circumscribed around the vertices of the flat face of the tetrahedron, is carried out based on the theorem on the projection of the right angle.

The result p^A (p_1^A, p_2^A) of the graphical solution of this subproblem is obtained, for example, for the face *BCD* (Fig. 6).

The algorithm for graphical construction of a complex drawing of the middle perpendicular includes the following three steps.

1. A complex drawing f^A (f_1^A, f_2^A) of the frontal f^A belonging to the face *BCD* is constructed.
   A complex drawing h^A (h_1^A, h_2^A) of the horizontal h^A belonging to the face *BCD* is constructed, if it was not constructed earlier.
2. A horizontal projection p_1^A of the middle perpendicular p^A to the flat face *BCD* of the tetrahedron *ABCD* is constructed through the horizontal projection O_1^A of the center O^A to be perpendicular to the horizontal projection h_1^A of the horizontal h^A.
3. A frontal projection p_2^A of the middle perpendicular p^A to the flat face *BCD* of the tetrahedron *ABCD* is constructed through the frontal projection O_2^A of the center O^A to be perpendicular to the frontal projection f_2^A of the frontal f^A (Fig. 6).

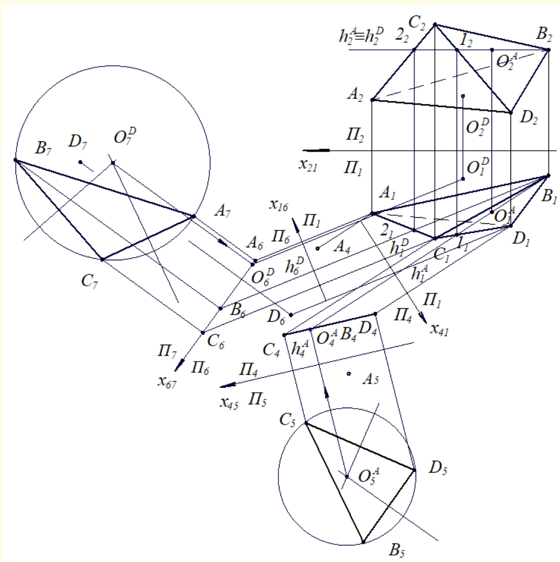

Figure 5: Complex drawings of centers O^A, O^D of circum-scribed circles for faces *BCD*, *ABC*.

Applying the described three-step algorithm, one can construct the middle perpendicular p^D to the face *ABC* adjacent to the face *BCD* (Fig. 6).

Complex drawings of the mid-perpendiculars p^B, p^C for the remaining faces *ACD*, *ABD* are constructed in a similar way.

The construction of a complex drawing K (K_1, K_2) of the point *K* of the intersection of the middle perpendiculars is performed as the intersection of the corresponding horizontal and frontal projections of any two perpendiculars.

On the complex drawings (Fig. 6, 7), the point K (K_1, K_2) of the intersection of the middle perpendiculars p^A, p^D to the faces *BCD*, *ABC* is developed (Fig. 7).

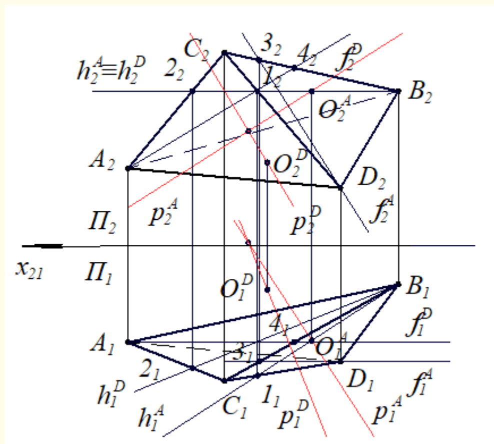

Figure 6: Complex drawings of the center perpendiculars p^A, p^D for the faces *BCD*, *ABC*.

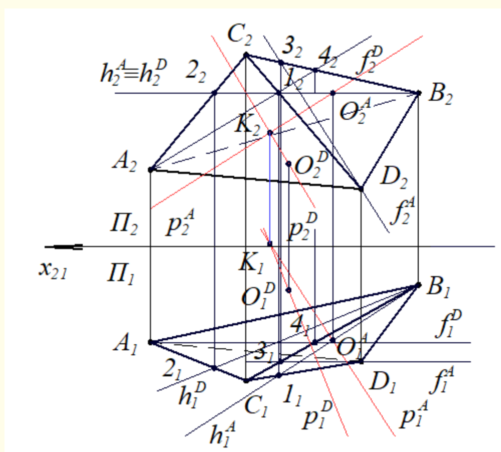

Figure 7: Proof of correctness of the correct graphic constructions of the complex drawing K (K_1, K_2) of the point *K*.

The same point K (K_1, K_2) of intersection of the middle perpendiculars p^B, p^C to the faces *ACD*, *ABD* can be constructed by determining the position of the centers O^B, O^C of the circumscribed circles for the faces *ACD*, *ABD*. Such graphic work was done and published [1-2].

The criterion of *correctness* of the graphical *constructions* presented in (Figs. 6, 7) is the fulfillment of the first law of projection connections according to which the constructed straight line connecting the horizontal K_1 and frontal K_2 projections of the K point is actually perpendicular to the abscissa axis [1-5].

The *truth* criterion of the obtained graphic *solution* is the equality of the values of the coordinates of the point K along the applicate axis for frontal projections and along the ordinate axis for horizontal projections for different pairs of perpendiculars p^A, p^D (Fig. 7) and p^B, p^C in various complex drawings [1-2].

Thus, the full validity of the second assumption about the presence of a point K of intersection of two mid-perpendiculars has been proved graphically.

Point K is equally distant from all four specified non-coincident points *A, B, C, D*, because it simultaneously belongs to the midpoint perpendicular p^A, all points of which are equally distant from points *B, C, D*, and it also belongs to the midpoint perpendicular p^D, all points of which equally distant from points *A, B, C*.

To verify the validity of the first assumption about the equidistance of the constructed point K from all four given non-coincident points *A, B, C, D graphically*, a complex drawing Σ (Σ_1, Σ_2) of the sphere Σ with center K is constructed, on the surface of which points *A, B, C, D* are located. It is, however, can be constructed if the radius of this sphere R_{HB} is known.

Using the method of a right-angled triangle [1-5] on a complex drawing of five points *A, B, C, D* and K, one can determine the true value of the radius R_{HB} of the sphere with points *A, B, C, D* equidistant from its center K as shown in Fig. 8.

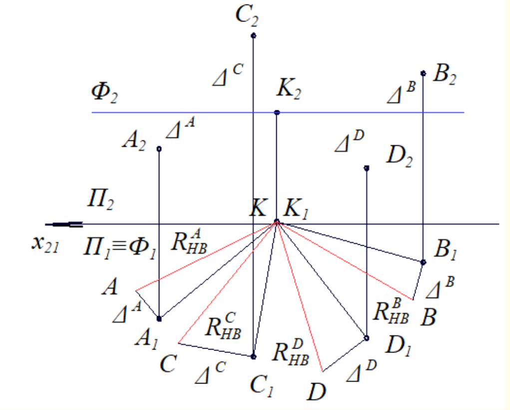

Figure 8: Determination of the actual value of the radius R_{HB} of the sphere.

The validity of the graphic constructions is confirmed by the equality of the true values of the radii $R^A{}_{HB}$, $R^C{}_{HB}$, $R^D{}_{HB}$, $R^B{}_{HB}$ of all four points *A, C, D, B* — $KA = KC = KD = KB = d_{tv}$.

The actual value of the radius R_{HB} of the sphere, to which all four points *A, B, C, D* do not coincide in the three-dimensional space, was determined graphically. $R_{HB} = R^A{}_{HB} = R^C{}_{HB} = R^D{}_{HB} = R^B{}_{HB} = d_{tv}$.

For a given spatial arrangement of points, *A, B, C, D* and a center *K* relative to these points, a sphere is constructed with a natural value of the radius R_{HB} equal to d_{tv} in a separate complex drawing shown in Fig. 9.

Visual construction of a sphere with center *K* and points *A, B, C, D* requires moving it away from the frontal plane and raising it above the horizontal plane as shown in Fig. 8.

The clarity of the complex drawing Σ (Σ_1, Σ_2) of the sphere Σ is achieved by moving the horizontal projections K_1, A_1, B_1, C_1, D_1 *of* the points *K, A, B, C, D* downward from the x21 axis bydt1and front projections K_2, A_2, B_2, C_2, D_2 points *K, A, B, C, D* upward from the x_{21} axis by d_{t2} (Fig. 9).

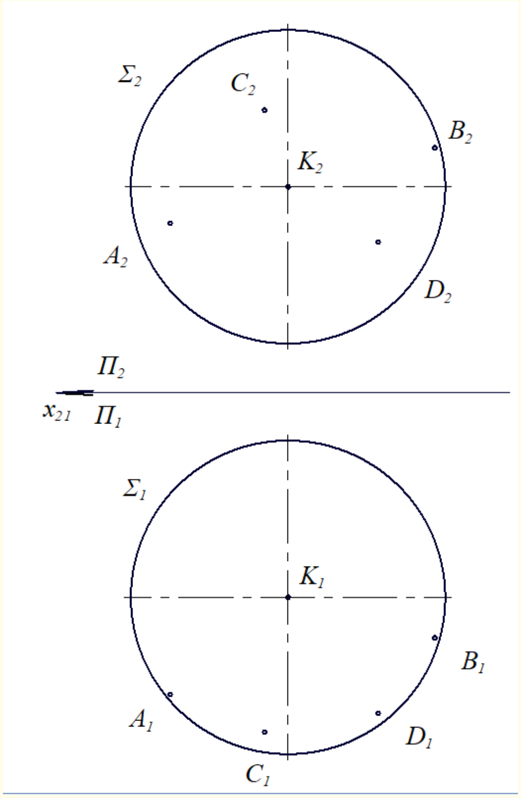

Figure 9: Complex drawing Σ (Σ_1, Σ_2) of the sphere Σ with points *A, B, C, D*.

To prove that points *A, B, C, D* belong to the surface of the sphere Σ, complex drawings of the intersection lines *a, b, c, d* of horizontally projecting planes Δ^A, Δ^B, Δ^C, Δ^D with the surface of the sphere Σ are constructed: $a=\Delta^A\cap\Sigma$, $b=\Delta^B\cap\Sigma$, $c=\Delta^C\cap\Sigma$, $d = \Delta^D\cap\Sigma$ as shown in Fig. 10.

As known, a point belongs to a surface if and only if it belongs to any line of this surface.

Because the frontal projection C_2 of the point *C* belongs to the frontal projection c_2 of the line *c* of the surface of the sphere Σ and the horizontal projection C_1 of the point *C* belongs to the horizontal projection c_1 of the same line c of the surface of the sphere Σ, then the point *C* itself belongs to the spherical surface, i.e., $C\in\Sigma$.

Similarly, it is graphically proved that point *D* belongs to the surface of the sphere Σ, since it is obvious from the drawing shown in Fig. 10 that the corresponding projections D_2, D_1 of point *D* belong to the corresponding projections d_2, d_1 of the line d of the intersection of the plane Δ^D with the surface of the sphere Σ.

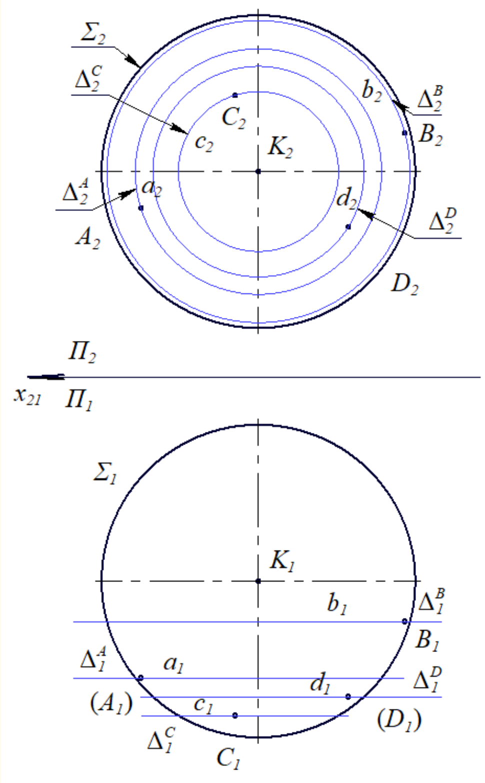

Figure 10: Proof of the truth of the obtained result.

The corresponding graphic proofs are carried out for points *A* and *B*.

Therefore, the point *K* is the locus equidistant from all four given non-coincident points *A, B, C, D*.

Thus, point *A* is located on the left front lower quarter of the surface of the sphere Σ, point *B* is located on the right front upper quarter of the surface, point *C* belongs to the left front upper quarter of the surface, and point *D* is located on the right front lower quarter of the surface of the sphere Σ centered at *K* (Fig. 10).

Since the points *A* and *D* are located on the front lower half of the surface of the sphere Σ, their frontal projections A_2, D_2 are visible, and the horizontal projections A_1, D_1 are invisible.

It is customary to enclose invisible projections of geometric images in parentheses – (A_1), (D_1).

Since points *C* and *B* are located on the front upper half of the surface of the sphere Σ, their frontal projections C_2, B_2 and horizontal projections C_1, B_1 are visible.

Methodology for the graphical solution of an engineering geometric problem

The proposed methodology for the graphical solution of an engineering geometric problem consists of a number of mandatory stages.

1. Analysis of the essence of the geometric phenomenon (Figs. 1-3).
2. Formulation of assumptions about the nature of the solution result.
3. Determination of the number of possible solutions to the problem.
4. Development of a method, technique and algorithm for a graphical solution (Figs. 4-9).
5. Verification of the validity of the obtained result (Fig. 10).

Conclusion

1. Based on the analysis of the *initial assumption*, it is proved that the locus of points equidistant from four points *A, B, C, D* that do not coincide in three-dimensional space is not the "axis of the cylinder, on the surface of which points *A, B, C, D* are located".
2. The performed research proves the validity of the assumption that the geometrical place of the desired image equidistant from four points *A, B, C, D* that do not coincide in the three-dimensional space is a *point*, denominated herein as *K*.Provided that any combination of three points out of the given four points does not belong to a straight line and a plane of level [3-5].
3. We have also proved the validity of the second put forward assumption that the center *K* of the spherical surface with points *A, B, C, D* is located at the intersection of the mid-perpendiculars restored to all four flat faces of the tetrahedron, the vertices of which *A, B, C, D* belong to the surface.
4. A method for solving a geometric problem is proposed, which consists in graphically constructing the centers of the flat faces of a tetrahedron, the vertices of which are located on a spherical surface, constructing mid-perpendiculars to each face through their centers and determining the point of intersection of these perpendiculars.
5. On the basis of the proposed method, a methodology for the graphical solution of an engineering geometric problem has been developed.

References

1. Brailov A Yu. "Methods for solving the Olympiad geometric problem". Modern problems of-modeling 18 (2020): 38-51.
2. Brailov A Yu. "Methodology for graphical solution of engineering geometric problems". Materials of the XXI International Conference on Mathematical Modeling. Kherson: KhNTU (2020): 90-91.
3. Brailov A Yu. "Engineering Graphics: Theoretical Foundations of Engineering Geometry for Design". Springer International Publishing (2016): 340.
4. Brailov A Yu. Engineering geometry [Tutorial]. K.: Caravel (2016): 472.
5. Brailov A Yu. Engineering geometry [Tutorial]. K.: Caravel (2021): 490.

Volume 3 Issue 6 December 2022