

Simulation with FlexSim an alternative to apply the M/M/C Model in a post COVID service system

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Abstract

Simulation with FlexSim an alternative to apply the M/M/C Model in a service system. *Introduction.* The study includes a literary review, model and simulation concepts, applications, the FlexSim characterization and the M / M / C model, that is, multiple channels. *Objective.* Address key concepts related to the use of FlexSim software within a simulation model in a service process where decisions can be made based on the study of queuing theory. *Method.* After performing the goodness-of-fit test for Poisson, it was determined that the arrival distribution to the line of people every hour does comply with a Poisson-type distribution because its Chi-square test reaches a value of 0.92 which represents that it is well above the coefficient of 0.5. Therefore, the exact probability of finding n arrivals during a certain time T can be found, if the process is random, as is the case of the cooperative, *Results.* The average number of clients in the queue waiting to be served, giving a reduction from 1.04 to 0.14 clients, so it is understood that, if the increase in servers in the cooperative were applied, this would cause that queues are generated in the system, since its L_q is 0.14 clients.

Keywords: Customers; Distribution; Model; Simulation; Queuing theory

Introduction

Queuing theory is a fundamental topic in Operations Research, as part of the approach to models that allow the efficient management of these lines, whether for products, people, materials, among others [1]. As part of a company's strategic plan, the customer's perspective and one of its main variables, "customer satisfaction", must be taken into account [2].

Queuing theory emerged at the beginning of the 20th century, when the problem of traffic congestion in telephone networks was first studied from a scientific approach by the Danish Agner Kraup Erlang. Since then, this theory has been applied to a multitude of real-life problems, such as those mentioned above [3]. It is a collection of mathematical models that describe waiting line systems. Such models serve to find a balance between the cost of the service and the cost associated with waiting for that service [4].

To describe the service system to be modeled, we used Kendall's extended notation, through which we specify: the probability distribution of the inter-arrival times, the probability distribution of the service times, number of servers, maximum number of simultaneous users allowed in the system, and queue discipline, respectively [7]. The simulation starts with a model. A model is a physical or mathematical description of a system and usually represents a particular point in time [8]. A system is a complex, integrated set of

interconnected elements, which is part of a higher-order system and is composed of higher-order systems [9]. The level of abstraction is difficult because most real systems are too complex for analytical evaluations, so systems must be studied by simulation [10]. In 2003, FlexSim software was released, which proved to be substantially different from previous simulators in both its simulation language and architecture [8].

The object under investigation is a small financial cooperative that analyses its current customer service queue in the post-COVID stage using operations management tools and statistics to facilitate decision making to reduce its operational expenses. Therefore, it seeks to answer the following questions: How would it affect the queue if I add one more customer service window and the arrival rate is expected to increase to more than 18 per hour? What effect will this have on the number of people in the queue?

Method

This Cooperative has a Poisson distribution for the arrival of customers, with an exponential service pattern, the queue has a PEPS discipline which means the first in is the first out, which are served by 2 employees. The FlexSim 2020 3D modelling allowed to analyse, visualise and improve the case to be solved in a suitable way, for its analysis. It also makes use of statistical tools such as: Goodness of fit test, chi-square test, exponential distribution and as a fundamental point of the work, mathematical modelling of waiting rows. This research was able to carry out 200 systematic observations.

Results

Characterisation of the service system

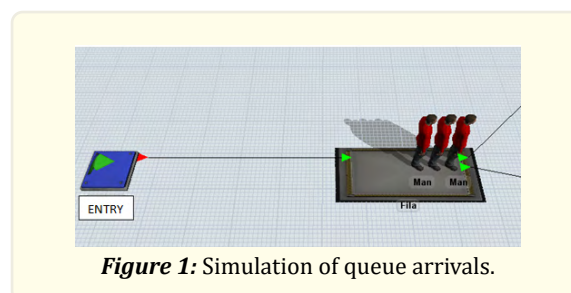
The starting point was to find out how the co-operative operates, customer behaviour and waiting times. Users arrive at the cooperative every day of the week during the cooperative's working hours (8:00 am to 16:00). (8:00 am to 16:00). As is usual for the type of queues that financial institutions generally manage, these are managed with an irregular timetable of arrival to the queue. The agency currently has 4 servers, which rotate in 2 shifts of 4 hours each, which means that they are always operating the 2 windows that are available to the public.

Arrival of customers

The cooperative's customer population is much larger in number than the number of servers at the counters, so for practical purposes it is considered to be an infinite population [11].

Distribution of arrivals

The arrival rate, which is the same as the number of units per period, must be established, so FlexSim 2020 software is used to simulate the counters and arrival times. As shown in figure 1.



By simulating the process, the following data base could be obtained regarding the number of customer arrivals at the queue, their waiting time and the time it takes to serve each customer. In a total of 67 hours, the number of customers arriving at the queue every hour is observed.

<i>N° Arrivals at the queue every hour</i>	<i>Frequency</i>
12	14
13	6
14	8
15	9
16	7
17	7
18	5
19	11

Table 1: Frequency of arrivals at the queue every hour.

It was determined whether the distribution of the hourly arrival time at the queue complies with a Poisson distribution, so a goodness-of-fit test for Poisson was performed, where the results in tables 2, 3 and 4 were obtained.

<i>N</i>	<i>Mean</i>
67	15,2687

Table 2: Descriptive statistics.

<i>N° Arrivals to the queue every hour</i>	<i>Probability of Poisson</i>	<i>Count observed</i>	<i>Count expected</i>	<i>Contribution to chi-square</i>
<=12	0,245951	14	16,4787	0,372855
13	0,092053	6	6,1675	0,004551
14	0,100395	8	6,7264	0,241135
15	0,102193	9	6,8469	0,677068
16	0,097521	7	6,5339	0,033244
17	0,087590	7	5,8685	0,218164
18	0,074299	5	4,9780	0,000097
>=19	0,199999	11	13,3999	0,429829

1 (12, 50%) of expected counts are less than 5.

Table 3: Observed and expected counts for N° Arrivals at the queue every hour.

Null hypothesis H_0 : The data follow a Poisson distribution.
 Alternative hypothesis H_1 : The data do not follow a Poisson distribution.

<i>GL</i>	<i>Chi-sq</i>	<i>p-Value</i>
6	1,97694	0,922

Table 4: Chi-square test.

Figure 1 shows the relationship between the observed values of the number of hourly arrivals at the cooperative's queue, where it can be seen that the highest expected value is found in the bar of less than 12 people per hour arriving at the queue.

After performing the goodness-of-fit test for Poisson, it was determined that, the distribution of hourly arrivals to the queue does meet a Poisson-type distribution because its Chi-square test reaches a value of 0.92 which represents that it is well above the coeffi-

cient of 0.5. So the exact probability of finding n arrivals during a given time T can be found, if the process is random as in the case of the cooperative, then the Poisson distribution is formulated as follows:

$$P_T(n) = \frac{(\lambda T)^n e^{-\lambda t}}{n!} \tag{1}$$

With all of the above said, we proceed to simulate in FlexSim 2020 and to take the time of arrival in the queue by customers at the customer service counters expressed in minutes, following a Poisson distribution, a total of 200 observations are taken. This results in the database shown in table 5.

3,09	3,27	4,93	4,05	3,83	4,91	4,5	3,25
3,19	3,38	3,92	4,89	4,3	4,45	3,05	3,36
3,83	4,34	4,83	3,29	4,28	4,87	4,4	3,15
4,4	3,68	4,82	4,22	4,18	4,21	3,88	4,96
4,97	4,56	3,1	3,48	4,12	4,75	3,76	3,3
3,97	3,07	4,91	4,99	3,38	4,52	3,08	3,96
3,5	3,58	4,93	4,02	3,83	3,71	4,51	4,52
4,91	4,18	3,35	3,83	3,75	3,48	4,5	3,17
4,94	4,73	4,08	4,95	4,67	4,04	4,41	4,07
4,47	4,96	4,01	3,16	4,09	3,5	3,95	4,5
3,27	4,29	3,14	4,61	3,57	4,95	3,52	4,87
4,2	4,19	3,84	4,45	4,61	3,16	4,96	4,64
3,79	3,55	3,59	4,41	4,39	4,5	3,08	4,46
3,57	3,57	4,4	4,0	3,84	4,41	3,43	4,76
3,53	3,06	4,96	4,1	4,36	3,49	3,26	3,69
4,06	3,68	4,79	4,08	4,16	3,79	4,76	3,21
3,92	3,05	3,4	3,32	4,6	4,16	3,03	3,07
4,55	3,33	4,77	3,31	4,15	3,61	3,57	4,29
3,91	4,35	4,26	3,11	3,97	4,29	4,21	4,82
3,23	3,72	4,43	4,61	4,81	3,76	3,34	3,36
4,81	3,38	4,09	3,33	3,1	4,43	4,9	4,97
3,82	4,66	3,01	3,69	3,99	3,14	4,71	3,04
3,59	4,98	3,49	4,14	3,98	4,95	4,26	4,19
3,79	4,32	3,24	4,61	3,36	3,04	4,86	3,89
3,75	4,63	4,59	3,06	3,86	3,34	3,53	4,66

Table 5: Customer arrival time at the queue (minutes).

For the simulation in FlexSim 2020 the properties for the simulation are expressed as shown in figure 2.

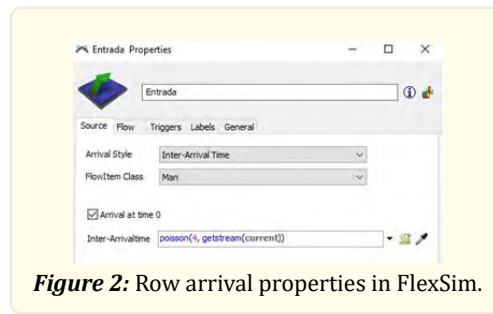


Figure 2: Row arrival properties in FlexSim.

Row system factors

Generally, institutions that provide financial services to natural persons work with a multi-channel, single-stage queuing structure, and the case of the credit union is no exception. This means that, given their asymmetric service time, customers are often not served equally fast; rather, some are served very quickly, depending on the service required, while others take much longer [11].

Distribution of service time

The cooperative has 2 counters operating throughout the 8-hour working day, so that, as explained above, its customer service time differs from customer to customer and, as is usually the case, an exponential distribution is used to simulate this type of case, as explained by Chase et al [11]. Therefore, the following data are obtained.

5,17	6,25	6,31	5,1	5,53	6,97	6,2	5,38
6,71	6,3	5,78	6,76	6,07	6,96	6,83	5,96
6,92	5,54	5,41	5,45	5,45	5,82	5	6,4
5,73	5,52	5,04	6,08	5,13	5,46	6,08	6,77
6,21	5,23	5,46	6,11	5,18	6,47	5,62	6,07
5,2	6,83	6,46	6,14	5,25	5,74	6,39	6,24
6,83	5,46	6,76	6,41	7,0	6,7	6,19	5,67
5,02	5,24	5,95	5,59	5,68	5,11	5,8	5,0
5,6	5,97	6,57	6,01	6,47	5,74	5,04	6,29
6,49	5,76	5,59	5,83	6,14	6,21	5,55	6,04
6,7	5,51	6,18	6,67	5,8	5,47	5,09	6,67
6,63	6,42	5,6	5,1	6,83	6,64	6,35	6,77
6,72	6,98	5,34	5,86	5,91	5,85	5,37	5,34
6,48	5,48	5,25	6,81	6,62	5,01	6,03	5,01
6,11	5,93	6,75	6,82	6,17	5,84	5,59	5,69
5,64	5,19	6,74	6,07	5,06	6,22	6,35	5,66
5,63	6,58	5,53	5,01	6,52	6,77	5,46	6,21
5,38	6,89	5,82	6,48	6,21	6,41	6,15	5,63
6,04	5,49	6,25	5,24	6,71	5,53	6,29	5,33
6,74	6,53	5,4	6,77	5,32	5,49	5,95	5,45
6,97	5,16	5,59	5,87	5,76	6,72	5,22	5,32
5,42	5,43	5,36	5,46	6,55	5,5	6,11	6,68

6,47	5,6	5,73	5,36	5,27	6,89	6,72	6,89
6,08	6,05	6,61	6,67	5,54	5,8	5,11	6,82
5,24	6,21	6,18	6,9	5,12	5,52	5,76	6,25

Table 6: Customer service time at the counter (minutes).

The sample mean is 5.9591 minutes.

For the simulation in FlexSim 2020, the properties for the simulation considering the above are expressed as shown in figure 3 in the case of counter service.

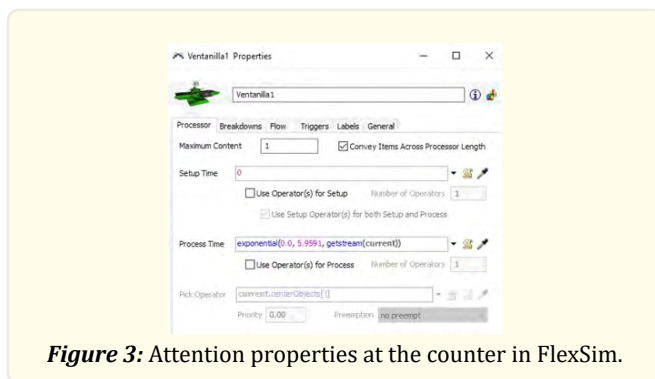


Figure 3: Attention properties at the counter in FlexSim.

Model M/M/C

The data obtained from the above study are as follows:

- $\lambda=0,222$ customers/minute (1)
- $1\lambda=4,496$ minutes (3)
- $\mu=0,222$ customers/minute (4)
- $1\mu=5,9591$ minutes (5)
- $c=2$ (6)

Where:

- λ : Average arrival rate in unit of time
- $1/\lambda$: Time between customer arrivals
- μ : Average service rate
- $1/\mu$: Service time
- c : Number of servers

The probability that there is no customer in the system was calculated using the following formula:

$$P_0 = \frac{1}{\left[\sum_{n=0}^{c-1} \frac{1}{n!} \left(\frac{\lambda}{\mu}\right)^n \right] + \frac{1}{c!} \left(\frac{\lambda}{\mu}\right)^c \frac{c\mu}{c\mu - \lambda}} \quad \text{for } c\mu > \lambda \tag{7}$$

$P_0=20, 28\%$

Once this was done, the average number of customers in the system was calculated.

$$L_s = \frac{\lambda\mu \left(\frac{\lambda}{\mu}\right)^c}{(c-1)!(c\mu - \lambda)^2} P_0 + \frac{\lambda}{\mu} \tag{8}$$

$L_s=2,36$ customers

The average number of customers in the queue waiting to be served is also calculated

$$L_q = L_s - \frac{\lambda}{\mu} \quad (9)$$

$L_q = 1,038$ customers

Once the L_s is obtained, the average time the customer spends in the system (waiting time plus service time) can be calculated.

$$W_s = \frac{L_s}{\lambda} \quad (10)$$

$W_s = 10,63$ minutes

The average time the customer spends in the queue is also calculated.

$$W_q = \frac{L_q}{\lambda} \quad (11)$$

$W_q = 4,67$ minutes

The cooperative intends to implement one more post COVID window in its customer service system, therefore, we proceeded to apply the M/M/C Model for 3 servers with the same data obtained previously, since it will be applied to the same financial entity, only the number of servers would change, which is the value of c . This is done so that the cooperative can observe what would happen if one more employee would be added to the customer service line and so the administrators of the cooperative have a back-up to make decisions.

$$\lambda = 0,222 \text{ customers/minute} \quad (12)$$

$$1/\lambda = 4,496 \text{ minutes} \quad (13)$$

$$\mu = 0,222 \text{ customers/minute} \quad (14)$$

$$1/\mu = 5,9591 \text{ minutes} \quad (15)$$

$$c = 3 \quad (16)$$

Resulting in the following: $P_0 = 25,65\%$

$L_s = 1,47$ customers

$L_q = 0,14$ customers

$W_s = 6,59$ minutes

$W_q = 0,63$ minutes

In order to compare how the cooperative's queuing system would look like if one more counter were added, the following comparison table was designed.

	2 servers	3 servers
P_0	20,28%	25,64%
L_s (customers)	2,36	1,47
L_q (customers)	1,04	0,14
W_s (minutes)	10,63	6,59
W_q (minutes)	4,67	0,63

Table 7: Comparison of results with one more server.

Conclusions

With the increase of one server in the cooperative's customer service system in the post-COVID stage, according to the M/M/C method there is an increase of approximately 5% in the probability that there will be no customers in the system compared to the waiting

line with 2 servers, which is logical since, if one more window is increased, this would cause the cooperative to run out of customers at some point since it supplies everyone.

On the other hand, the average number of customers in the system was reduced to 1.47 customers. This is also the case for the average number of customers in the queue waiting to be served, because it was practically reduced to zero customers, so it is understood that, if the increase of servers in the cooperative were applied, this would cause no queues to be generated in the system, since its L_q is 0.14 customers.

The average time that the client spends in the system was reduced from 10.63 minutes to 6.59 minutes due to the increase of one more server, the same happened with the average time that the client spends in the queue, since this decreased considerably from 4.67 minutes to 0.63 minutes, this happens because the client population is well supplied by the servers since 3 customer service windows would be enabled and therefore no queues would be generated and if there were, it would be in a minimum amount and with a very low waiting time.

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