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Research Article

Acceleration Analysis of a Rotating Object

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Abstract

Any rotating object with variable velocity about a fixed point generates centrifugal and tangential accelerations that are the components of the inertial forces. The known textbooks of engineering mechanics contain chapters for the acceleration of a rotating object that is presented by not accurate mathematical processing. Such a mathematical model for the acceleration of the rotating object yields the incorrect result that leads an unjustified increase in the size of the mechanical component of machines. In engineering, rotating components of machines is a subject of exact computing of inertial forces which non-observance reflects on the reliability of their work. This work considers an analysis of the acceleration of a rotating object at a fixed point and presents the precise mathematical model for its acceleration.

Keywords: Radial and Tangential Acceleration; Rotating Object; Fixed Point

Introduction

All known publications in engineering mechanics contain the topic of analysis of acceleration of the rotating mass about the fixed axis. The mass of the rotating object with variable spinning generates radial and tangential inertial forces. Conditions of the high spinning of the object lead to increase inertial forces and destruction of the mechanism that is inadmissible in engineering. The analysis of the chapters dedicated to the acceleration of the rotating objects in textbooks in classical mechanics demonstrates the vagueness in their mathematical processing in analytical approaching [1-3]. The textbooks in theoretical and engineering mechanics consider the common case of the accelerated rotation for the object about a fixed point. The expression of the common acceleration presented the radial acceleration at the condition of a constant angular velocity, but the tangential acceleration presents at the condition of the variable angular velocity [4-7]. Such wrong formulation of tangential and radial accelerations for rotating mass gives incorrect expressions and incorrect results in computing of inertial forces acting on mechanisms [8-11]. The chapters of acceleration for a rotating object in textbooks of physics and dynamics of mechanical systems contain the component of acceleration with the instant angular velocity. This presentation of the common acceleration (Eq.1) does not have a strong validation for such an analytical approach that unacceptable for the exact sciences [12, 13].

$$\alpha = r\sqrt{\varepsilon^2 + (\omega_{in} + \varepsilon t)^4} \qquad (1)$$

where r is the radius of rotation of the object about a fixed point, ε is the angular acceleration, ω_{in} is the initial angular velocity, t is the time.

The one publication presents a correct physical formulation for the acceleration of the rotating object but contains a processing error of the mathematical model that gives incorrect expression [14].

Information about inaccuracy in the analytical solutions for the acceleration of the rotating objects presents the challenge for the physicists and mathematicians that should be resolved finally. Incorrect computing of inertial forces acting in mechanisms results in the worsening of their technical parameters and their quality. Absence of the exact methods for calculating forces acting on machines in engineering dos does not give to experts to evaluate the perfectness of technics.

Following analysis of accelerated rotation of the object about the fixed point covers the gap in the theoretical study of dynamics in engineering mechanics.

Methodology

An accelerated motion of a rotating object about a fixed point presented in Figure 1 is typical in classical mechanics. Figure 1 demonstrates the rotating object *p* linked with the radius *r* about the fixed-point *o* on the *xoy* plane. The object *p* rotates with the angular velocity ω and acceleration ε. The diagram presents for consideration the vectors of the initial and final tangential velocity V_{in} and V_{fn} of the object p respectively that separated by an angle α.

The vectorial velocity polygon of the rotating object *p* is solved for the change in the tangential velocity, $V_c = V_{fn} - V_{in}$, where the vector V_c is presented by the line *bc*, V_{fn} by the line *ac*, and V_{in} by the line *ab* respectively.

The variable tangential velocity V_{fn} is expressed by the following equation:

$$V_{fn} = r\omega_{in} + \varepsilon rt \qquad (2)$$

where $V_{in} = r\omega_{in}$, ω_{in} is an initial angular velocity of a rotating point, and εrt is an extra velocity of a moving point due to an acceleration one, other parameters of equation are as specified above.

The vector V_r of radial velocity is presented by the line bd and vector V_t of tangential velocity by the line *dc*. The vector V_r is directed toward the point o of rotation and crosses the vector V_{fn} at the point *f*. Then the vector V_r is expressed by the sum of two vectors $V_{r.bf}$ and $V_{r.fd}$, of segments *bf* and *fd* respectively (Figure. 1), $V_r = V_{r.bf} + V_{r.fd.}$ The vector of the change in the tangential velocity V_c can be presented by the following vectorial expression: $V_c = V_r + V_t$.

The acceleration of a rotating object about a fixed point can be defined by three physical components that are its tangential velocity $V = \omega r$, tangential acceleration, αt, and radial one α_r.

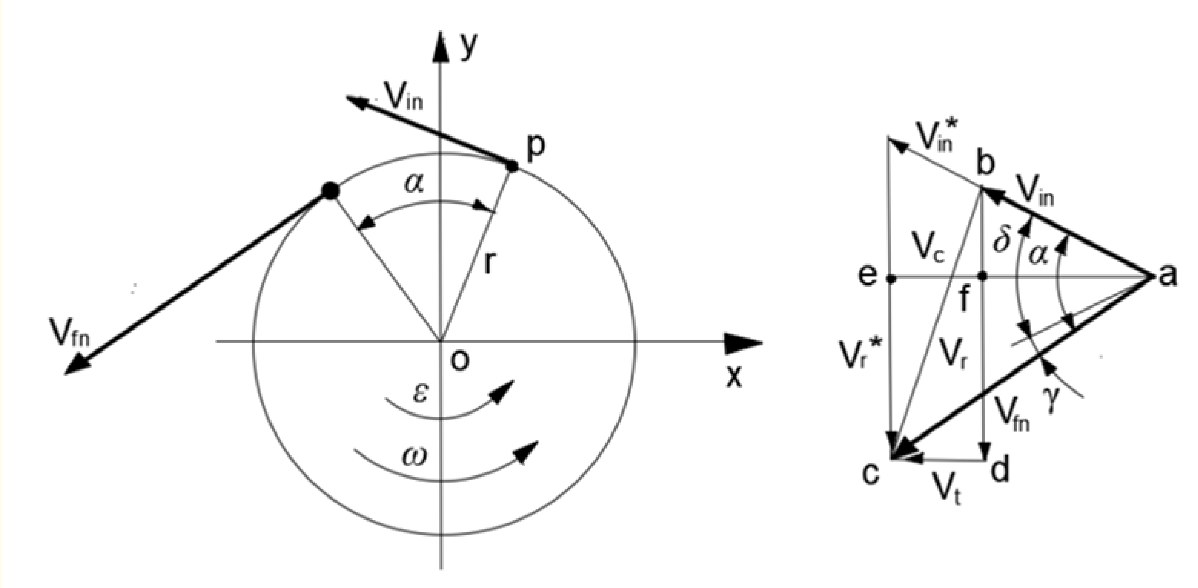

Figure 1: Vectorial velocity polygon of the rotating object about a fixed point.

The small angle α of the rotation of the object p (Figure 1) presents the sector that has the following expression: $\alpha = \omega_{in} t + \frac{\varepsilon t^2}{2}$. The angular sector α can be presented by the sum of angles $\alpha = \delta + \gamma$, the angle δ is expressed by an initial angular velocity $\delta = \omega_{in} t$ and angle γ by accelerated angular velocity $\gamma = \alpha - \delta = \frac{\varepsilon t^2}{2}$, where t is the time, other components are as specified above. The two components of the radial velocity V_r can be presented by the following expressions:

$V_{r.bf} = V_{in} \sin(\delta/2)$ and $V_{r.fd} = V_{fn} \sin(\delta/2 + \gamma)$, where the vector $V_{r.fd}$ is presented by the segment $fd = ec$.

Obtained expressions enable to yield the following equation of the radial velocity for the rotating object p.

$$V_r = V_{in} \sin(\delta / 2) + V_{fn} \sin(\delta / 2 + \gamma) \qquad (3)$$

The trigonometric identity for the small angel $\sin\varphi = \Delta\varphi$ enables to simplify of Eq. (3)

$$\Delta V_r = V_{in} (\Delta\delta / 2) + V_{fn} (\Delta\delta / 2 + \Delta\gamma) \qquad (4)$$

where expressions $V_{in} = \omega_{in} r$, $V_{fn} = \omega_{in} r + \varepsilon r t$, $\Delta\delta = \omega_{in} \Delta t$, and $\Delta\gamma = \varepsilon \Delta t^2 / 2$ are defined above.

Defined parameters are substituted into Eq. (4) and transformation yield the following equation:

$$\Delta V_r = \omega_{in} r \frac{\omega_{in}\Delta t}{2} + (\omega_{in} r + \varepsilon tr)\left(\frac{\omega_{in}\Delta t}{2} + \frac{\varepsilon \Delta t^2}{2}\right) = \omega_{in}^2 r \Delta t + \frac{\omega_{in} r \varepsilon \Delta t^2}{2} + \frac{\varepsilon tr\, \omega_{in}\Delta t}{2} + \frac{\varepsilon^2 tr\, \Delta t^2}{2} \quad (5)$$

Equation (5) is presented by the differential form in which the first derivative concerning time yields the radial acceleration of the rotating object about a fixed point

$$\frac{dV_r}{dt} = \frac{\omega_{in}^2 r}{2}\frac{dt}{dt} + \frac{\omega_{in}^2 r}{2}\frac{dt}{dt} + \frac{\omega_{in} r\varepsilon}{2}\frac{dt^2}{dt} + \frac{\varepsilon tr\,\omega_{in}}{2}\frac{dt}{dt} + \frac{\varepsilon^2 tr}{2}\frac{dt^2}{dt} = \omega_{in}^2 r\frac{dt}{dt} + \omega_{in} r\varepsilon t\frac{dt}{dt} + \frac{\omega_{in} r\varepsilon t}{2}\frac{dt}{dt} + \varepsilon^2 t^2 r\frac{dt}{dt} \quad (6)$$

where all components are as specified above.

Simplification of Eq. (6) yields the expression of the radial acceleration of the rotating object.

$$a_r = r\left[\omega_{in}^2 + \frac{3}{2}\varepsilon t\omega_{in} + (\varepsilon t)^2\right] \quad (7)$$

The absolute acceleration of a rotating object about a fixed point is obtained by the square root of the tangential a_t and the radial a_r accelerations that have the following expression:

$$a = \sqrt{(\varepsilon r)^2 + \left\{r\left[\omega_{in}^2 + \frac{3}{2}\varepsilon t\omega_{in} + (\varepsilon t)^2\right]\right\}^2} = r\sqrt{\varepsilon^2 + \left[\omega_{in}^2 + \frac{3}{2}\varepsilon t\omega_{in} + (\varepsilon t)^2\right]^2} \quad (8)$$

Analysis of Eqs. (7) and (8) demonstrates the considerable difference from the similar equations presented in textbooks. Equations (7) and (8) give the less values of the radial and absolute accelerations. The new equations of accelerations for rotating object about fixed point enable to reduce the weight and size of a mechanism in engineering.

Case study

The rotating object (Figure1) is running with the initial angular velocity of 10 rad/s, accelerates at the rate of 7 rad/s^2 and has a radius of rotation of 0.1 m. Determine the values of the radial and absolute accelerations for the object after 8 seconds of rotation.

Solution.

The radial acceleration by Eq. (7) is as follows:

$$a_r = r\left[\omega_{in}^2 + \frac{3}{2}\varepsilon t\omega_{in} + (\varepsilon t)^2\right] = 0{,}1\times[10^2 + (3/2)\times 7\times 8\times 10 + (7\times 8)^2] = 407{,}6 \quad m/s^2$$

The radial acceleration is component of Eq. (1) of textbooks [1-3] is as follows:

$$\alpha_r = r(\omega_{in} + \varepsilon t)^2 = 0{,}1 \times (10 + 7 \times 8)^2 = 435{,}6 \quad m/s^2$$

The absolute acceleration by Eq. (8) is as follows:

$$a = r\sqrt{\varepsilon^2 + \left[\omega_{in}^2 + \frac{3}{2}\varepsilon t \omega_{in} + (\varepsilon t)^2\right]^2} = 0{,}1\sqrt{7^2 + \left[10^2 + (3/2) \times 7 \times 8 \times 10 + (7 \times 8)^2\right]^2} = 407{,}600 \qquad m/s^2$$

The absolute acceleration is by Eq. (1) of textbooks [1-3] is as follows:

$$a = r\sqrt{\varepsilon^2 + (\omega_{in} + \varepsilon t)^4} = 0.1\sqrt{7^2 + [10 + 7 \times 8]^4} = 435{,}600 \quad m/s^2$$

The results obtained for the radial and absolute accelerations of a rotating object by new Eqs. (7) and (8) give the less value than by Eq. (1) of textbooks [1-3]. Practitioners and researchers should take such a big difference in results into account because it leads to a decrease in the dimensions and weights of the machine parts and increases their quality of work.

Results and Discussion

The computing of acceleration for the rotating object is important for the reliable work of a machine. The known mathematical models for the accelerations of the rotating objects about a fixed point have vagueness in the analytical approach and their application in engineering does not warrant correct results. Conducted mathematical analysis of the acceleration yields the exact equations for accelerations of the rotating object around a fixed point. The obtained mathematical model for the acceleration gives the correct result in computing and less value in inertial forces acting on mechanisms. This result is important for practitioners that reduces the weight of the machine components and universities should use it in the educational process of machine dynamics.

Conclusion

The conducted analysis of the accelerations for the rotating objects about a fixed point enables deriving the fundamental equations for the dynamics in mechanics. The work presents new mathematical models for the radial and absolute accelerations of a rotating object. The new one giving exact results in computing should replace the simplified models for accelerations of rotating objects in the textbooks and manual. The new acceleration analysis of a rotating object around a fixed point should be used in textbooks of machine dynamics and by practitioners in engineering. In that regard, this is also a good example of educational processes.

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