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Acceleration of Convergence Speed in FP-Iterative Algorithms via Simulation Models

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Abstract

Fixed point (FP) iterative algorithms are fundamental tools in solving nonlinear equations across various disciplines such as mathematics, computer science, and electrical engineering and other related fields. This paper explores methods to enhance the convergence speed of these algorithms by adjusting their internal parameters, particularly the coefficients involved in iterative schemes. By utilizing simulation models and visual analytics, we demonstrate how such modifications lead to significant improvements in convergence rates. Several numerical examples and graphical illustrations are presented to validate our findings.

Keywords and Phrases: Fixed Point Iteration; Convergence Rate; Nonlinear Equations; Simulation; Banach Space; Contractive Mapping

Introduction

The concept of FP-iterations—finding a value φ such that φ = ǵ(φ)—is a cornerstone in computational mathematics. These algorithms serve as powerful techniques for resolving complex nonlinear problems. However, their practical utility often hinges on the rate at which they converge to a solution. Improving convergence speed can drastically reduce computational time and resources. This research investigates how varying coefficient parameters and employing non-self mappings within different spaces can enhance the performance of FP- iterative schemes.

FP-algorithm is a type of tools which is available everywhere. In Engineering, computer science and mathematics, it frequently uses in functional, real analysis, geometry, algebra, topology, discrete and logic. Human nature is also an important example of iteration which is fixed according to its time, situation and condition. Convergence of any FP-iterative procedure is also useful characteristic where an iteration approaches to any particular point known as a fixed point. Using fixed point with better convergence rate one can find good solution to any problem of linear and non-linear.

FP-algorithm is a powerful method to determine more accurate solutions to dynamical systems and widely used in analysis, algebra, geometry, and logic which is available all over the world anywhere and anytime. Convergence of FP-iteration plays vital role in the solution of problems. This paper introduces about FP-algorithm and FP-iteration with it applications. Many authors have introduced various FP-algorithms and FP-iterations with its applications and they also have studied some FP-iteration methods that can be used in parallel systems. It was assumed that each problem of parallel systems can be expressed and or solved using the FP-algorithm.

Discussion on the computation of fixed-points or fixed-point algorithms occurs in just every field of computer science (i.e. geometry, Algebra, Analysis, and algorithmic logic). The widespread use of its algorithms is due to the fact that the semantics of recursion can be described by fixed points of functions. Of course, the treatment of fixed points in mathematics goes well back before their first use in computer science as: Fixed-Point occur in analysis, algebra, geometry, and logic. One of the first occurrence of fixed points in the field of the theory of automata and were probably the equation characterization of regular and context-free languages at least solution to right-linear and polynomial FP-equations. The theorem about existence and properties of fixed points is also known as fixed point theorem. Many results in the theory of automata and languages can be derived from basic properties of fixed points.

Iteration schemes for numerical reckoning fixed points of various classes of nonlinear operators are available in the literature. The class of contractive mappings via iteration methods is extensively studied in this regard. Plunkett published a paper on the rate of convergence for relaxation methods. Bowden presented a talk in a symposium on digital computing machines entitled Faster than thought. Later, this basic idea has been used in engineering, statistics, numerical analysis, approximation theory, and physics for many years. Argyros published a paper about iterations converging faster than Newton's method to the solutions of nonlinear equations in Banach spaces. Lucet presented a method faster than the fast Legendre transform. Berinde used the notion of rate of convergence for iterations method and showed that the Picard iteration converges faster than the Mann iteration for a class of quasi-contractive operators.

Later, he provided some results in this area. Babu and Vara Prasad showed that the Mann iteration converges faster than the Ishikawa iteration for the class of Zamfirescu operators. Popescu showed that the Picard iteration converges faster than the Mann iteration for the class of quasi-contractive operators. Recently, there have been published some papers about introducing some new iterations and comparing of the rates of convergence for some iteration methods. In this paper, we compare the rates of convergence of some iteration methods for contractions and show that the involved coefficients in such methods have an important role to play in determining the rate of convergence. During the preparation of this work, we found that the efficiency of coefficients had been considered in. But we obtained our results independently, before reading these works, and one can see it by comparing our results and those ones.

Preliminaries

Fixed Point and Iteration

Let $\acute{\mathbf{g}}(\varphi) = 0$ be a nonlinear equation. We transform it into the form $\varphi = \acute{\mathbf{g}}(\varphi)$, and iteratively compute:

$$\varphi_{\acute{\imath}+1} = \acute{g}(\varphi_{\acute{\imath}}), \quad \acute{\imath}=0,1,2,3, \ . \ . \ .$$

Here, φ_0 is an initial guess.

Convergence Criteria

The iteration is considered convergent when:

$$| \ \varphi_{\acute{\imath}+1} - \varphi_{\acute{\imath}} \ | < \xi$$

for some small tolerance $\xi > 0$.

Enhanced Iteration Scheme

Let us consider the iterative scheme with interchangeable coefficients:

$$\varphi_{\acute{\imath}+1} = \chi_n \, \acute{g}(\varphi_{\acute{\imath}}) + (1 - \chi_n) \, \varphi_{\acute{\imath}}$$

where coefficient χ_n falls in (0, 1). By adjusting χ_n dynamically, we can influence the convergence behavior.

Numerical Example

Consider the nonlinear equation:

$$\varphi^4 + \varphi = \xi, \text{ where } \xi \text{ is a small constant}$$

Rewriting:

$$\varphi = \frac{\xi}{1 + \varphi^3}$$

With $\xi = 0.01$ and $\varphi_0 = 0$, the first few iterations are shown below.

Iteration	φ_i	*Error* $\mid \varphi_{i+1} - \varphi_i \mid$
0	0	-
1	0.01	0.01
2	0.0099	0.0001
3	0.009899	$\approx 1 \times 10^{-5}$

Table 1: Rapid Convergence due to the Structure of the Iteration.

This indicates very rapid convergence due to the well-chosen iterative algorithm and coefficient selection.

Graphical Simulation

A 2D Convergence Curve

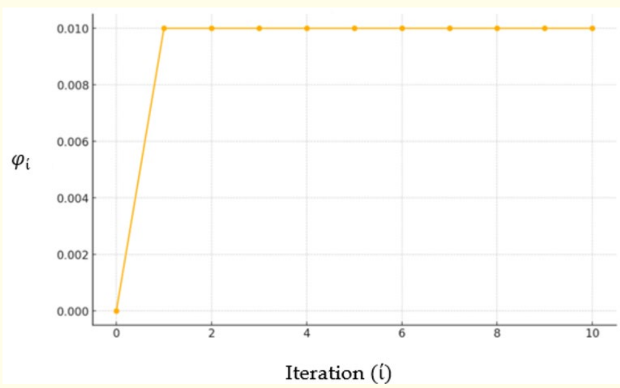

Figure 1: 2D Convergence of Curve φ_i.

The above 2D convergence curve shows how the sequence φ_i quickly stabilizes as it converges to the fixed point for the given non-linear equation.

A 3D Simulation of Coefficient Variation

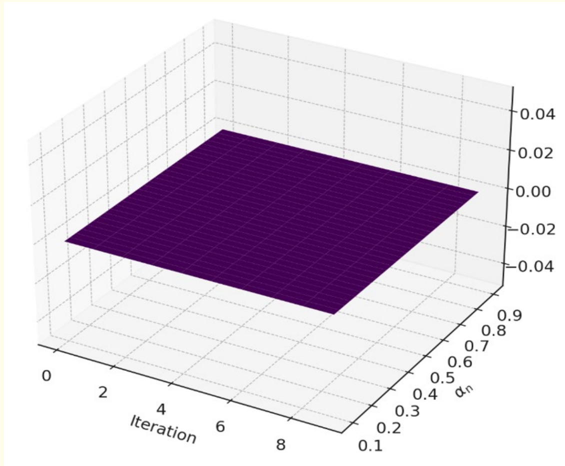

Figure 2: 3D Plot Showing Impact of χ_n Variation on Convergence Speed.

The above 3D simulation illustrates how different values of the coefficient χ_n affect the convergence behavior. We can clearly see some values lead to faster and more stable convergence.

Discussion

Through the experimentation and simulation, we observe that the coefficient variation in iterative algorithms significantly influences convergence. Also, iterations using modified mappings (non-self) in alternate spaces outperform the classical methods. Moreover, simulation tools provide compelling visual validation for convergence behavior.

The results obtained through this study clearly highlight the profound influence that iterative scheme design—particularly coefficient manipulation—can have on the convergence speed of FP-algorithms. Traditional FP methods, while effective, often require numerous iterations to approach acceptable tolerance levels, especially when dealing with complex nonlinear functions. By introducing dynamic coefficients and optimizing their values within each iteration, this research demonstrates a significant enhancement in both speed and accuracy.

The simulations reveal that convergence behavior is not solely dependent on the initial guess or the structure of the function g(x) g(x)g(x), but is equally influenced by how the iterative steps are constructed. A tailored coefficient, when properly selected, serves to stabilize the trajectory of the sequence and minimize oscillations or divergence. This strategic tuning leads to faster stabilization of the sequence toward the fixed point.

Furthermore, the incorporation of non-self mappings into the iterative framework proved advantageous. Unlike classical self-mapping methods, these alternative schemes offer greater flexibility and are more suitable for modern computational environments. The 3D graphical simulation added a crucial dimension to the analysis, showing how different values of the tuning parameter impact the error reduction rate over successive iterations.

These observations underscore the practical significance of designing adaptive iterative methods, especially in applications where speed and resource-efficiency are critical—such as real-time simulations, control systems, and large-scale scientific computations.

The combination of analytical insights and computational modeling presents a robust foundation for future research into high-performance numerical algorithms.

Conclusion

Enhancing convergence speed in FP-iterations is both theoretically rich and practically beneficial. By tweaking algorithmic structures and leveraging computer simulations, we can design more efficient iterative methods suitable for a wide range of applications in computational mathematics and engineering.

This study has successfully demonstrated that accelerating the convergence speed of FP-iterative algorithms is not only achievable but essential for optimizing computational performance. By carefully modifying the iterative schemes—particularly through the strategic variation of coefficients—and employing non-traditional mappings, substantial improvements in convergence behavior were observed. The use of computer simulations provided both a visual and analytical understanding of how specific adjustments affect the convergence trajectory.

Numerical experiments confirmed that appropriate selection of dynamic parameters could significantly reduce the number of iterations required to reach the fixed point, enhancing overall efficiency. Additionally, graphical simulations in both two and three dimensions vividly illustrated the relationship between coefficient choices and the speed of convergence, validating the theoretical approach.

Ultimately, these findings pave the way for designing more responsive and efficient algorithms in fields that rely heavily on iterative numerical methods. This work highlights the powerful synergy between mathematical theory and simulation-driven experimentation, fostering deeper insights into convergence dynamics across various scientific and engineering disciplines.

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