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Random Oscillations of Nonlinear Dynamical Systems

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Real dynamical systems tend to contain individual elements with nonlinear characteristics. Such characteristics correspond to a number of elements:

- limiters, as well as devices with saturation of magnetic characteristics in electronic and electromagnetic systems;
- Rubber parts, vibration dampers, as well as leaf springs in mechanical systems , etc.

Therefore, when studying oscillations in such systems, it is necessary to take into account the nonlinearity and characteristics of individual elements and the features of oscillations inherent in nonlinear systems with such characteristics [13, 30]. The most significant features of nonlinear oscillatory systems are as follows:

- They do not apply the principle ***of superposition***;
- Several equilibrium positions may exist in them;
- Their free oscillations ***are notizochronous***, that is, the frequency of free oscillations depends on the initial conditions;
- They are ***ambiguous***, that is, the results of the solution depend not only on the frequency, but also on the amplitude of the perturbations; it is possible to appear oscillations with frequencies of 2, 3, etc. times lower frequencies of the main oscillations - ***subharmonic oscillations***, as well as with frequencies of 2, 3, etc. times higher frequencies of the main oscillations - ***superharmonic*** (***ultraharmonic***) ***oscillations***.
- The possibility of ***the appearance of self-oscillatory modes***, i.e. non-extinguishing oscillations even in the absence of an active perturbation, despite the presence of dissipative forces.

In addition, in nonlinear systems, ***the effects of capture, tightening, vibration linearization***, etc. may appear.

There are no general methods for the analytical study of oscillations of any nonlinear systems, although with some types of nonlinear dependence between the members of the differential equation of oscillations of a nonlinear system with one degree of freedom, it is possible to reduce the solution to the calculation of quadratures. In most cases, the motion characteristics of such systems are obtained by using special approximate methods, and now on the basis of numerical integration.

Often, the study of the dynamic properties of a crew system is usually reduced to the study of virtual models with a finite number of freedom under the action of stationary random processes of perturbationy. At the same time, although they take into account the nonlinearities of the system, they do not study their effect on the motion features of such nonlinear systems in sufficient depth and do not identify nonlinear effects inherent in these systems. This is a consequence of the fact that that in the study of oscillations of nonlinear systems, the same technique is used as in the study of linear ones. At the same time, they are given a perturbation in the form of a single implementation of a multidimensional random process, implicitly assuming that random fluctuations in generalized co-ordinates are stationary and even ergodic.

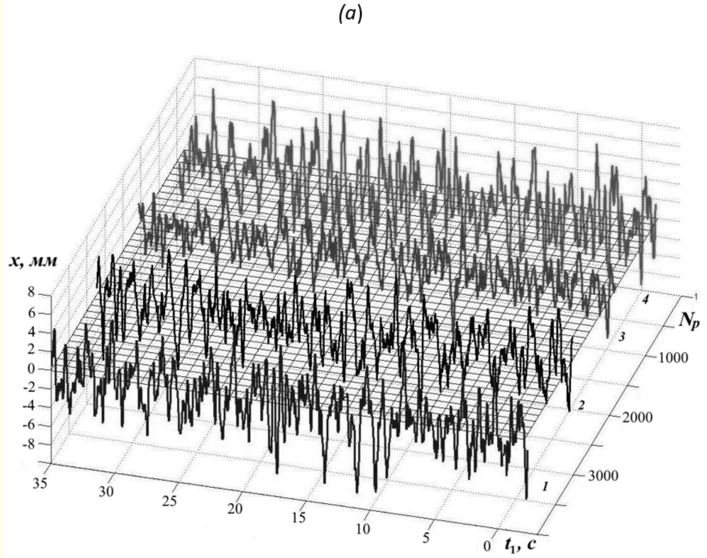
However, with this approach, the choice of perturbation implementation is random and, therefore, the results of the calculations are also random. If you use another perturbation implementation of many such implementations, the results of the calculations will be different. In addition, when solving systems of nonlinear differential equations, ***it is necessary to use implicit numerical integration schemes that do not require lowering the order of the differential*** equations, that is, bringing the system of nonlinear differential equations to the Cauchy form. Such schemes include $\boldsymbol{\beta}$ - Newmark diagram, Park diagram, etc., available in all modern software packages.

Since the amplitudes of the forced oscillations of a nonlinear system ambiguously depend on the frequency and amplitude of the perturbation, its random oscillations will be non-stationary. Therefore, in order to determine the probabilistic characteristics of such ***non-stationary*** oscillations, it is necessary to average many implementations of the random oscillation process [3, 11, 12, 15]. To obtain such a set, it is necessary to generate many implementations of perturbation processes on a computer. At the same time, the number of points in each $\boldsymbol{N}$ perturbation implementation and the number of such implementations $\boldsymbol{N_p}$ should be the same and contain at least $\boldsymbol{2^{12} = 4096}$ values for the possibility of performing spectral analysis using the “fast Fourier transform” (FFT) algorithm.

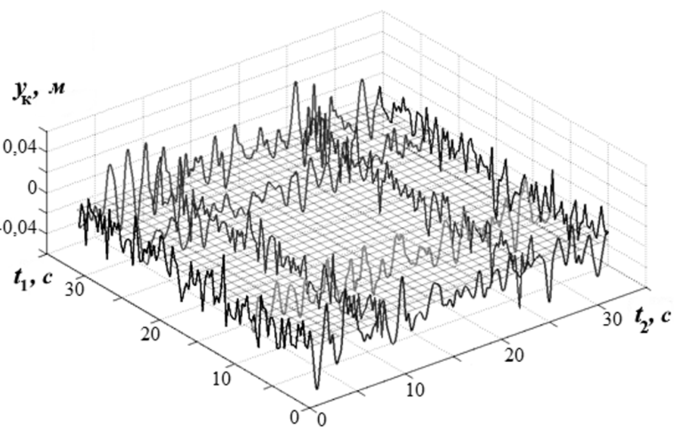
For example, consider the results of the calculation of random oscillations of the rail crew with nonlinear characteristics of the spring suspension. Graphs of random perturbation implementations in the form of geometric path irregularities $\boldsymbol{n(t)}$ (Fig. 1, a) show that these processes have the form of random oscillations with stable deviations from the zero mean value. Therefore, the implementations $\boldsymbol{n(t)}$ can, to a first approximation, be considered as implementations of an ergodic stationary random process.

In addition, those individual implementations of the random process of plane-parallel $\boldsymbol{y_k(t)}$ transverse oscillations of the crew body (deviation oscillations) (Fig. 1, b) differ significantly from each other, as ***the scope*** of random oscillations, i.e., the scale of random oscillations. ***Both dispersion and frequency composition***, which confirms the ***non-stationarity of*** these random oscillation processes. To obtain the probabilistic characteristics of a non-stationary random process, $\boldsymbol{y_k(t)}$ it is first necessary to determine the two-dimensional probability density $\boldsymbol{f[y_k(t_1), y_k(t_2), t_1, t_2]}$, which in this case corresponded to Gauss’s law of distribution for independent random arguments.

(a)



b)

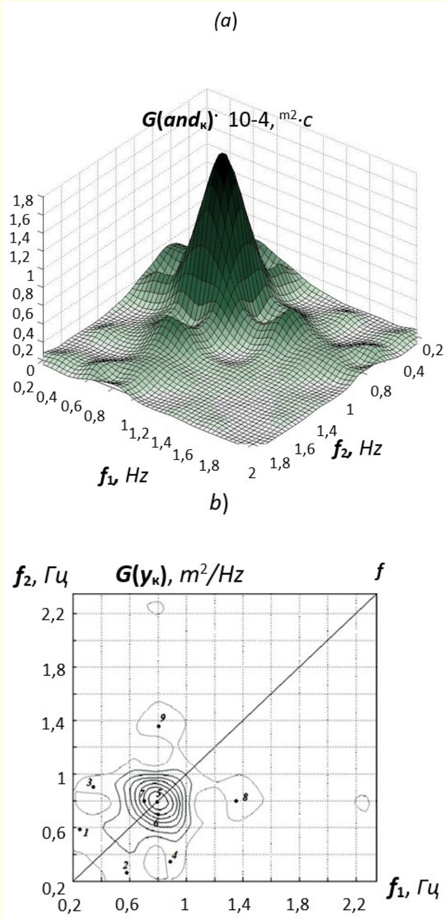


Rice 5.3: Implementation of stationary (a) and non-stationary (b) random processes ac-cording to geometric unevenness of the track and vibrations of the body of the electric train car at a speed of 20m / s (72km / h) Replace *x η*(t) with !!!.

Knowing the two-dimensional probability density, it is possible to find the correlation function of a non-stationary random process $R_{y_к}(t_1,t_2)$ and by this function the spectral density, $G_{y_к}(f_1,f_2)$ the graph of which (Fig. 3) so far suggests that the location of the maxima of this function is symmetrical with respect to the bisectrix passing through the origin. The greatest maxima of spectral density correspond to the basic frequencies oscillations of the linearized system, lie on the main diagonal and $Of\ G_{Ki}^{\max}$ fall on the frequencies of the s $f_{1-1}=0,25Гц$; and $f_{4-4}=0,38Гц$ $f_{7-7}=0,70Гц$. The frequencies of lateral maxima have the following approximate ratios:

$$\frac{f_{2-1}}{f_{1-1}}\approx\frac{0,59}{0,25};;\ \frac{f_{4-1}}{f_{4-4}}\approx\frac{0,88}{0,38}\ \frac{f_{8-1}}{f_{7-7}}\approx\frac{1,35}{0,70}.$$

Here, the first number of the index corresponds to the number of the point on the diagonal (Fig. 6.1, b), and the second to the value of the frequency along the axes or this figure. In other words, these frequency ratios are close to the ratio of 2: 1. Given that the amplitudes of the lateral maxima are less than the amplitudes of the main ones located on the diagonal, it can be considered that in this nonlinear system $f_1\,f_2$ ***ultraharmonical*** oscillations occur at frequencies exceeding by ~2 or more times the frequencies of the main maxima.



Rice 6.1: Spectral density of non-stationary random oscillations body weight at a speed of 20m/s: *a* – view from the end of the frequency axes; *b* – top view.

Findings

1. Random oscillations of nonlinear systems are non-stationary.
2. For probabilistic analysis of non-stationary random oscillations of nonlinear systems, it is necessary to use an averaging algorithm for a set of implementations of the studied random process.

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