

The Link Between the Spread Information and the Maxwell's Equation

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Abstract

The Forces between two charges are one the four fundamental forces that keep the world together Indeed when a single photon is emitted, it spreads over a system, the intensity decrease by square law of the distance which compliant with conservation of energy. A changing magnetic field creates an electric field, and a changing electric field creates a magnetic field, and an accelerating electric field creates an accelerating magnetic field. Electric lines are modeled for visualizing the effect in space of electrostatic forces that occur two or more charged particles. Maxwell equation provides a Mathematical model for electric, optical and radio technologies such as power generation, electrical motor and wireless communication. Unfortunately, Maxwell's equation requires some advanced techniques which lie well beyond the understanding of this paper. The derivation of maxwell's equations is approach established for the time -dependent from static laws, The variation of electrical fields and magnetic field and Faraday's law can be derived without any assumption relativistic.

Keywords: Mathematical model; Maxwell's equations; Electromagnetism; Wireless communication

Introduction

From understanding the level of relativity, we can actually explain the existence of magnetic fields as just electric fields that classical-ly shouldn't exist but do as a consequence of length contraction. These circumstances are identical to those of a classic magnetic field and force, if you replace the rods with a neutral current carrying wire. In this case, the dog is the positive charge experience a force of repulsion due to the same charge [1, 3].

Assuming the conducting wire carrying a current, protons are still they are not moving only electrons move because you know they are free to move but protons reside. In the field line in what the equation was recognized. Maxwell was the first person to use this equation to explain that light is an electromagnetic phenomenon, counting the number of field lines in a surface closed. Therefore, it is related to electric flux in any closed Gaussian surface that enclose the electric charge. The correction of Ampere's law and Maxwell's states that magnetic fields can be generated in two ways: by electrical current this was the original Ampere's law and by changing electric fields this was Maxwell's correction represented by the following equation: [9].

$$F = qE + q \vec{V} \times B \quad \dots 1$$

Then equation (1) is called the foundation of classical of electromagnetism and electrodynamics, which are branch of theoretical physics studying the interaction between electric charge and current using the classical Newtonian Model for objects governed by classical mechanics, if the present state is known, it is possible to predict in the future determinist and how it has moved in the past [10].

Using the standard Heaviside notation, the standard Heaviside notation, and assuming that the conservation of charge of Coulomb's law in electrostatics and Ampere's law, The Electric charges are sources of electric fields. Changing magnetic fields can also create electric fields. The electromagnetic force on a charge q is a combination of a force in the direction of the electric field E which is proportional to the magnitude of the field and the quantity of charge. The force at right angles of magnitude field B and the velocity v of the charge are also proportional to the magnitude of the field and the velocity [1-2].

Derivations of Maxwell's equations using the Heaviside notation

Maxwell's equations can be found throughout his 1861 paper, derived theoretically using a molecular vortex Model of Michael Faraday's lines of force in conjunction result of weber Kohlrausch. In 1884, a work with similar work of Willard Gibbs and Heinrich Hertz grouped the four equation into different set called as Hertz-Heaviside equation, Maxwell-Hertz equation [14].

The concept of fields was introduced by Faraday, according to Albert Einstein, the precise formulation of the time-space laws was Maxwell's work after taking some decades to accept and understand his discovery which was genius upon the conception of his colleagues [15]. The work of Heaviside was to eliminate the electrical potential and magnetic potential used by Maxwell as the principal concepts of his equations. This effort was not well accepted because was controversial, so in 1884 was clear to understand that the potential must propagate at the speed of light. According to the modern analysis from radio antennas, makes use of Maxwell's vector and scalar potentials to separate the variables. With common techniques used in formulating the solution of the differential equations, in which the potential can be introduced by algebraic solution and manipulation of the four fundamental equations [16].

In memory of 125 years of Oliver Heaviside's work showing the restructuration of the original twenty Maxwell 'equations to four equations were recognized as Maxwell's equations developed in Mathematical form. The introduction of electromagnetism show that the Maxwell' equations are true because of the overwhelming body of experimental data that validates them, not only does not describe the electric-field and Magnetic-field from charges and current in a vacuum but by considering the charges and currents produced in materials .Heaviside was the engineer employed in the British post office telegraph system in Newcastle, was the one who took the Maxwell 'equations from what eliminate the vector and scalar potential and developed the differential vector notation in calculus ,written them down in the form we currently use. Heaviside's form gives us the Electric-field and the magnetic-field they can produce and opens the way to describe wave and energy propagation more directly [3, 4, 14].

Maxwell's theory was accepted and advanced by others, particularly by Oliver Heaviside, Heinrich Hertz and Hendrik Lorentz. Today Heaviside is a champion of the Faraday-Maxwell approach equations in electromagnetism by simplifying Maxwell's original set of twenty equations to the four used today. Heaviside rewrote Maxwell 'equations in a form that involves only electric and magnetic fields. In an analogy to gravity, the field corresponds to the gravitational force pulling an object onto the Earth, while the potential corresponds to the shape of the landscape on which it stands. Heaviside simplified Maxwell's equation to duplex notation by configuring them in term of fields through evident symmetry developing the mathematical subject of vector of calculus with which to apply the equations. The analysis of the interaction of electromagnetic wave with conductors, derived from the telegrapher's equations of Kirchhoff from Maxwell's theory to describe the propagation of electrical signal along a transmission line in 1888 Hertz made his most significant contribution with the discovery of radio waves [16].

In the 20th century, Maxwell's equations had an impact beyond electromagnetism in the discovery of the theory of relativity and the field equations of quantum mechanics, which is the profound and the most fruitful in experimental physics since the time of Newton the time of quantum mechanics though less clearly the link to electromagnetism. The theory of Maxwell is based on the concept of invisibility all penetrating medium through which the electromagnetic fields propagate. While in 1892 after Maxwell's death, the equations remain valid in the description of all electromagnetic phenomena. The maxwell's equations are used in the design of all types

of electrical and electronic equipment. The reason why these equations can be solved only for structures of high symmetry [17, 18].

The derivative of Time-dependent differential equation

The derivative of time-dependent differential equation for what the sound is propagating it can be found using the Hooke's static law of elasticity using in description of static equilibrium in gas state represented by the following formula:

$$p - p(0) = \frac{B}{\rho_D(0)} (\rho_D - \rho_D(0)) \quad \dots 2$$

Where B is the bulk modulus $p(0)$ and $\rho_D(0)$ representing the initial pressure and density of the gas and p is the applied pressure and ρ_D the resultant density. Then the Hooke's static law can written as:

$$\left(\frac{\partial p}{\partial \rho_D} \right) = \frac{B}{\rho_D(0)} \quad \dots 3$$

Equation 2 relates how a change in the external pressure can be applied to a uniform and static gas changes the density through the entire system, while the equation 3 describes how a differential pressure across an infinitesimal volume can be the responsible of a differential change in density [5].

It describes the gas as being in state of equilibrium so that even though the pressure and density can vary as a function of space and time. The local of equilibrium ensures that the differential of pressure concerning space or time is related to an equivalent differential for density, represented by the following formula:

$$\left(\frac{\partial p}{\partial \rho} \right)_x = \frac{B}{\rho_D(0)} \left(\frac{\partial \rho}{\partial t} \right)_x \quad \dots 4$$

However it is important to notice that it is not possible to derive equation (2) or (3) from 1 using mathematics methods only but it is derived from the equation that describe the propagation of sound, represented by using Newton's second law of motion we have:

$$\left(\frac{\partial p}{\partial x} \right)_t = - \frac{\rho^2 D(0)}{B} \left(\frac{\partial \mu}{\partial t} \right)_x \quad \dots 5$$

Where μ is the velocity and t the time and by using identity, we have:

$$\left(\frac{\partial p}{\partial x} \right)_t = \left(\frac{\partial p}{\partial \rho_D} \right)_t \left(\frac{\partial \rho_D}{\partial x} \right)_t \quad \dots 6$$

And equation (2) of Newton's law gives the following:

$$\left(\frac{\partial \rho_D}{\partial t} \right)_x = - \rho D(0) \left(\frac{\partial u}{\partial t} \right)_t \quad \dots 7$$

From conservation of mass, we have:

$$\left(\frac{\partial \rho_D}{\partial t} \right)_x = - \rho D(0) \left(\frac{\partial u}{\partial t} \right)_t \quad \dots 8$$

Doing partial differential from equation (2) with respect to t and combining to partial derivative with respect to x of equation (6) and allowing changes in the order of differentiation we find the following:

$$\left(\frac{\partial^2 u}{\partial t^2}\right)_x = \frac{B}{\rho D(0)} \left(\frac{\partial^2 u}{\partial t^2}\right)_t \quad \dots 9$$

From equation (8) where the velocity of sound v is given by:

$$v^2 = \frac{B}{\rho D(0)} \quad \dots 10$$

The Hooke's static law extension to the time domain allows us to understand a whole new range of phenomena associated with pressure waves. Then the derivatives of these laws, the differential equations with respect to time and space for the whole system. The main idea of this approach looks to make the system sufficiently providing the general differential equations [6].

Derivations of Maxwell's four equations

The divergence of Electric field

According to the generalization of coulomb's law of electrostatics, to a time-dependent form where the point of observation and the charges present are local and is represented by:

$$E(r, t) = \lim_{\eta \rightarrow 0} \frac{1}{4\pi\epsilon_0} \int \frac{\vec{\eta}}{\eta^2} \rho'(r', t) d\tau \quad \dots 11$$

Where the electric field E at a point of observation p located at a point $r(x, y, z)$ and time t is produced by charge densities $\rho'(r', t)$ located at primed points $r'(x', y', z')$ at the same time t . By definition $\eta = r - r'$ and $d\tau'$ denotes integrating over the primed spatial variable of the charge densities while the unprimed spatial variable remain constant. By assuming that all the charge are local and very close to the point of observation then the partial time derivative of the electric field E at the point of observation is given by:

$$\frac{\partial E}{\partial t} = \lim_{\eta \rightarrow 0} \frac{1}{4\pi\epsilon_0} \int \frac{\vec{\eta}}{\eta^2} \frac{\partial \rho'}{\partial t} d\tau' \quad \dots 12$$

Where $\frac{\partial \rho'}{\partial t}$ is calculated at time t which is a standard convention, the partial derivatives with respect to time are calculated assuming that all the spatial variable are primed and unprimed and are held, by definition of the del operator ∇ we have the following:

$$\nabla = \left(\frac{\partial i}{\partial x}\right)_{yz} + \left(\frac{\partial j}{\partial y}\right)_{xz} + \left(\frac{\partial k}{\partial z}\right)_{xy} \quad \dots 13$$

The operator, in addition to the unprimed spatial variable that are explicitly held constant, for each of the partial derivatives, the variable t and the primed variable x', y' and z' are also held constants the the equations (11) and (13) can be written as:

$$\nabla \cdot E = \lim_{\eta \rightarrow 0} \frac{1}{4\pi\epsilon_0} \int \frac{\vec{\eta}}{\eta^2} \frac{\partial \rho'}{\partial t} d\tau' \quad \dots 14$$

The charge densities and current densities are displaced from the origin at the point r' . The vector separation between the charge density ρ', r' or current density $j'(r', t)$ and the observation point represented by $\eta = r - r'$. The electric and magnetic field at the primed location are E and B , respectively. In the special case of the electric field, charge density, magnetic field and current density at the point of observation it is recommended to use the unprimed values E, ρ, B and j respectively also applying the following identities we have:

$$\nabla \cdot \left(\frac{\vec{\eta}}{\eta^2} \right) \rho' = \rho' \nabla \cdot \left(\frac{\vec{\eta}}{\eta^2} \right) + \left(\frac{\vec{\eta}}{\eta^2} \right) \cdot \nabla \rho', \nabla \cdot E = \frac{\rho}{\epsilon_0} \text{ and } \nabla \cdot \left(\frac{\vec{\eta}}{\eta^2} \right) = 4\pi\delta^3 \quad \dots 15$$

From the above identities where ρ only depends on primed variable and the time t then we obtain one of the Maxwell's equations which is the following:

$$\nabla \cdot E = \frac{\rho}{\epsilon_0} \quad \dots 16$$

However, the derivative was done from the local time-dependent equation and by assuming that the fundamental differential equations that is correct at all points in space and time the above formula is called Maxwell's first equation [7].

The divergence of magnetic field

By applying the Ampere's law of magnetism and invoking the ocean's razor to postulate that the local time-dependent of magnetic field B at the time t is represented by:

$$B(r, t) = \lim_{\eta \rightarrow 0} \frac{1}{4\pi} \int j'(r', t) x \frac{\vec{\eta}}{\eta^2} d\tau \quad \dots 17$$

Where j' is only a function of the primed spatial variable and the time t assuming also that the equation (16) is only valid for a system where all the current densities are local to the point of observation.

$$\frac{\partial B}{\partial t} = \lim_{\eta \rightarrow 0} \frac{1}{4\pi} \int \frac{\partial j}{\partial t} x \frac{\vec{\eta}}{\eta^2} d\tau \quad \dots 18$$

Trying to use a general vector field identity the equation (16) can be written as follows:

$$\nabla \cdot \left(j' x \frac{\vec{\eta}}{\eta^2} \right) = \left(\frac{\vec{\eta}}{\eta^2} \right) \cdot (\nabla x j) - j' \cdot \left(\nabla x \frac{\vec{\eta}}{\eta^2} \right) \quad \dots 19$$

since $\nabla x j' = 0$ and j' is printed, the divergent lead to

$$\nabla \cdot B = \frac{\mu_0}{4\pi} \int X - j' \nabla \frac{\vec{\eta}}{\eta^2} d\tau' \quad \dots 20$$

Using the vector field identity $\nabla x \frac{\vec{\eta}}{\eta^2} = 0$ lead to the second of Maxwell's equation which is:

$$\nabla \cdot B = 0 \quad \dots 21$$

The equation (21) has the same form and is similar by using Mathematical identities to derive the standard result from magneto-statics. The local time-dependent was derived by assuming that it is correct at all points in space and time [8].

The curl of magnetic field

The Maxwell's displacement current density and by taking the curl of both sides of Ampere's law we have the the following:

$$\nabla \times B = \mu_0 J \text{ if } \nabla \cdot (\nabla \times B) = 0 \text{ then } \nabla \cdot J = -\frac{\partial \rho}{\partial t} \quad \dots 22$$

Since the equation (20) cannot be generally true as a function of time and given the vector field identity $\nabla \cdot (\nabla \times B) = 0$ also by invoking the idea of continuity of charge from the equation given by $\nabla \cdot J = -\frac{\partial \rho}{\partial t}$ and considering the partial time derivative of $\nabla \cdot E = \frac{\rho}{\epsilon_0}$ we have the third equation of Maxwell represented by:

$$\nabla \times B = \mu_0 J + \mu_0 \frac{\partial B}{\partial t} \quad \dots 23$$

Where the current density flows in wires to charge capacitor plates and produces a changing of electric field E between the plates and invoking the Stroke's theorem, we can write that $\nabla \times B = \mu_0 J$. Which can be represented in term of the line integral of the magnetic field around the path that surrounding the plates that bounds surface A and the surface integral across surface is the following:

$$\oint B \, dl = \mu_0 \int J \, ds \quad \dots 24$$

The right hand side of the equation (22) is $\mu_0 \int J \, ds$ representing the surface A and zero for the surface B because no current passing through surface B. To ensure that the line integral of B does not depend on whether surface A or surface B. Knowing that the current density is given by:

$$J = \epsilon_0 \frac{\partial E}{\partial t} \quad \dots 25$$

Where E is the electric field between the plates then the Maxwell 's displacement current density in term of $\nabla \times B = \mu_0 J$.

$$\mu_0 J \quad \dots 26$$

To produce the Maxwell's brilliant equation which led to the unification of electricity and magnetism:

$$\nabla \times B = \mu_0 J + \mu_0 \frac{\partial B}{\partial t} \quad \dots 27$$

Maxwell's Third equation

Using the derivative of the Maxwell's third and fourth equations and assuming that the system is constraint by the conservation of charge that implies the following:

$$\frac{\partial \rho'}{\partial t} = -\nabla \cdot J' \quad \dots 28$$

Where J' and ρ' are the current density and charge density at point r' and using the definition.

$$\nabla' = \left(\frac{\partial i}{\partial x'} \right)_{y'z'} + \left(\frac{\partial j}{\partial y'} \right)_{x'z'} + \left(\frac{\partial k}{\partial z'} \right)_{x'y'} \quad \dots 29$$

This operator, is similarly to what presented above in addition to the primed spatial variable explicitly, the variable t and the non-primed variables x, y and z are also held constant by substituting the equation (23) into the following:

$$\frac{\partial E}{\partial t} = \lim_{\eta \rightarrow 0} \frac{1}{4\pi\epsilon_0} \int \frac{\partial J}{\partial t} x \frac{\vec{\eta}}{\eta^2} d\tau \quad \dots 30$$

And then using a standard vector field that includes changing the order of partial derivatives and the vector field

identity $\nabla \cdot \left(\frac{1}{\eta} \right) = -\frac{\vec{\eta}}{\eta^2}$ we have the following expression:

$$\frac{\partial E}{\partial t} = \lim_{\eta \rightarrow 0} \frac{1}{4\pi\epsilon_0} \nabla \cdot \int \nabla' J' d\tau \quad \dots 31$$

This is a vector field algebraic used in magnetostatics as a technique for function of three spatial variables that can be used and rearranged the right-hand side of the equation (26) and using the following identity.

$$\nabla' \left(\frac{1}{\eta} \right) J = \frac{1}{\eta} \nabla' \cdot J' + \nabla' \left(\frac{1}{\eta} \right) \cdot J' \quad \dots 32$$

and integrating (27) we have the following:

$$\int \frac{1}{\eta} \cdot \nabla' J' d\tau' = \int \nabla' \left[\frac{1}{\eta} J' \right] d\tau' - \int \nabla' \left[\frac{1}{\eta} \right] J' d\tau' \quad \dots 33$$

With $J=0$ and $\nabla' \left[\frac{1}{\eta} \right] = -\nabla' \left[\frac{1}{\eta} \right]$ we have the following expression:

$$\frac{\partial E}{\partial t} = \frac{1}{4\pi\epsilon_0} \nabla \int \nabla' \left(\frac{1}{\eta} \right) \cdot J' d\tau' = \frac{1}{4\pi\epsilon_0} \nabla \int \nabla' \left(\frac{1}{\eta} \right) \cdot J' d\tau' = \frac{1}{4\pi\epsilon_0} \nabla \int \nabla' \left(\frac{J'}{\eta} \right) d\tau' \quad \dots 34$$

Trying to use the vector field for curl of a vector field $\nabla^2 \left(\frac{1}{\eta} \right) = -4\pi\delta^3(\eta)$, the vector field identity for the curl of the product of a vector field and scalar $\nabla X J' = 0$, and $\nabla' \left[\frac{1}{\eta} \right] = -\frac{\eta}{\eta^2}$ we have the following situation:

$$\begin{aligned} \frac{\partial E}{\partial t} &= \frac{1}{4\pi\epsilon_0} \int \nabla^2 \left(\frac{J'}{\eta} \right) d\tau' \\ &+ \frac{1}{4\pi\epsilon_0} \nabla \int \nabla X \left(\nabla X \frac{1}{\eta} \right) d\tau' = \frac{1}{4\pi\epsilon_0} \int J' \nabla^2 \left(\frac{1}{\eta} \right) d\tau' - \frac{1}{4\pi\epsilon_0} \nabla \int \nabla X \left(\frac{\eta}{\eta^2} X J' \right) d\tau' = -\frac{J}{\epsilon_0} \\ &+ \mu_{0\epsilon_0} \nabla X B \\ \nabla X B &= \mu_0 J + \mu_{0\epsilon_0} \frac{\partial E}{\partial t} \quad \dots 35 \end{aligned}$$

Which is the third Maxwell equation.

Discussion

Comparing the mathematical approach used for the derivative of Maxwell's approach and considering a steady- state of the system. It was invoked the Ocean's razor for the generalization of coulomb's law by finding the expression for $\frac{\partial E}{\partial t}$...34

The curl of Electric field

By starting with faraday's law which is:

$$\nabla X E = 0 \quad \dots 36$$

It describes that the measure of the force on stationary charges and using loops of metallic wire carrying unbound stationary charges attached to voltmeters-The faraday's experiment together with Lenz's experiment described mathematically as:

$$\nabla X E = K \frac{\partial B}{\partial t} \quad \dots 37$$

where k is a constant of proportionality approximately equal to unity [9].

Maxwell's fourth equation

Let us consider the primed partial time derivative of the following expression:

$$\nabla' X \frac{\partial B'}{\partial t} = \mu_0 \frac{\partial J'}{\partial t} + \mu_{0\epsilon_0} \frac{\partial^2 E'}{\partial t^2} \quad \dots 38$$

Where B' , J' and E' are the magnetic field, the current density, and the electric field at the point r' , and using the vector field identity we the following:

$$\nabla' X(\nabla' X E') - \nabla'(\nabla' \cdot E') + \nabla'^2 \cdot E = 0 \quad \dots 39$$

Which gives us the expression:

$$\frac{\partial B}{\partial t} = \frac{1}{4\pi} \int \left\{ \nabla' X \left(\frac{\partial B'}{\partial t} + \nabla' X E' \right) \right\} + \left[\nabla'^2 \cdot E' - \nabla'(\nabla' \cdot E') - \mu_0 \varepsilon_0 \frac{\partial^2 E'}{\partial t^2} \right] X \frac{\vec{\eta}}{\eta^2} d\tau' \quad \dots 40$$

Also invoking the vector field identity which is:

$$\nabla' X \left(\frac{\rho' \eta}{\eta^2} \right) = \rho' \cdot \nabla' X \left(\frac{\vec{\eta}}{\eta^2} \right) + \nabla'(\rho') X \left(\frac{\vec{\eta}}{\eta^2} \right) \quad \dots 41$$

Knowing that the curl of radial function is zero, and the second term of the above equation is also zero hence using the vector field identity represented by:

$$\nabla' \left[\frac{\rho'}{\eta} \right] = \left(\frac{1}{\eta} \right) \nabla'(\rho') + \rho' \nabla' \left(\frac{1}{\eta} \right), \nabla' \left[\frac{1}{\eta} \right] = -\frac{\eta}{\eta^2} \quad \dots 42$$

We found the following term:

$$\int \nabla(\nabla \cdot E) X \frac{\eta}{\eta^2} d\tau' = \frac{1}{\varepsilon_0} \nabla X \int \frac{1}{\eta} \nabla'(\rho') d\tau' + \frac{1}{\varepsilon_0} \nabla X \int (\rho') \frac{\vec{\eta}}{\eta^2} d\tau = -\frac{1}{\varepsilon_0} \int \nabla' X \nabla' \left(\frac{\rho'}{\eta} \right) d\tau' = \frac{1}{\varepsilon_0} \int \nabla X \nabla' \left(\frac{\rho'}{\eta} \right) d\tau \quad \dots 43$$

Following the same procedure using the vector identity we have:

$$\nabla' \left[\frac{\rho'}{\eta} \right] = \left(\frac{1}{\eta} \right) \nabla'(\rho') + \rho' \nabla' \left(\frac{1}{\eta} \right), \nabla' \left[\frac{1}{\eta} \right] = -\frac{\eta}{\eta^2} \quad \dots 44$$

And gives us the following expression:

$$\int \nabla(\nabla \cdot E) X \frac{\eta}{\eta^2} d\tau' = \frac{1}{\varepsilon_0} \nabla X \int \frac{1}{\eta} \nabla'(\rho') d\tau' + \frac{1}{\varepsilon_0} \nabla X \int (\rho') \frac{\vec{\eta}}{\eta^2} d\tau \quad \dots 45$$

Applying Coulomb's law and Ampere's law and ignoring the internal structure of any element of charge and current density and by assuming that the volume occupied by every element of charge density and current density are negligible and the second integral equal to zero we have:

$$\int \nabla'(\nabla' \cdot E') X \frac{\vec{\eta}}{\eta^2} = 4\pi \nabla X E \quad \dots 46$$

Calculating the derivative in term of time t .

$$\frac{\partial B}{\partial t} = \frac{1}{4\pi} \int \left\{ \nabla' X \left(\frac{\partial B'}{\partial t} + \nabla' X E' \right) \right\} + \left[\nabla'^2 E - \mu_0 \varepsilon_0 \frac{\partial^2 E}{\partial t^2} \right] X \frac{\vec{\eta}}{\eta^2} d\tau - \nabla X E \quad \dots 47$$

And combining all together we have the following expression:

$$\nabla X E = -\frac{\partial B}{\partial t} \quad \dots 48$$

Taking the above Maxwell's equations in the primed form together using the mathematical the following identity

$$\nabla' X (\nabla' X E') - \nabla' (\nabla' \cdot E') + \nabla'^2 \cdot E = 0 \dots 49$$

And using the derivate of electric field E propagating through vacuum that $\rho = 0$ and $J' = 0$ we have the form:

$$\nabla'^2 E' - \mu_0 \epsilon_0 \frac{\partial^2 E}{\partial t^2} = 0 \dots 50$$

The given Maxwell's first three equations, equation 41 is the solution to the following expression:

$$\frac{\partial B}{\partial t} = \frac{1}{4\pi} \int \left\{ \nabla' X \left(\frac{\partial B'}{\partial t} + \nabla' X E' \right) \right\} + \left[\nabla'^2 E - \mu_0 \epsilon_0 \frac{\partial^2 E}{\partial t^2} \right] X \frac{\vec{\eta}}{\eta^2} d\tau - \nabla X E \dots 51$$

Putting this set of equation aside as non-physical because they imply that any change in charge density or current density would instantaneously change the electric field and the magnetic field throughout the entire universe. It is the fourth Maxwell's equations. However, Faraday's law can be derived without any relativistic assumption about Lorentz's invariance. The general solution to Maxwell's four equation is:

$$E(r, t) = \frac{1}{4\pi\epsilon_0} \left\{ \int \frac{\vec{\eta}}{\eta^2} \rho' r d\tau + \int \frac{\vec{\eta}}{\eta^2} \frac{\partial \rho'}{\partial t} r d\tau - \int \frac{1}{\eta c^2} \frac{\partial J'}{\partial t} d\tau \right\} \dots 52$$

$$B(r, t) = \frac{\mu_0}{4\pi} \left\{ \int J X \frac{\vec{\eta}}{\eta^2} d\tau' + \int \frac{\partial J'}{\partial t} r X \frac{\vec{\eta}}{\eta^2} d\tau \right\} \dots 53$$

Faraday 'law describes how a time-varying magnetic field induces an electric field. This electromagnetic induction is the operating principle behind many electric generators. However, it is necessary to comply with the laws of Gauss. The theory of electromagnetism is based on Maxwell's equations in the local and integral form. The change in the electric field in time is dependent on the change in magnetic field across space. Iterating the electric field and H-field updates result changing in time process [10, 50].

Conclusion

Maxwell's equations describe how electric charges and electric currents act as sources for the electric fields. It also describes how time -varying electric field generates a time-varying magnetic field and vice versa. One can be observed from the mathematical description of the laws mentioned in the paper. From the four equations, two of them, Gauss's law for magnetism, describes how the field emanates from charges. For the magnetic field, there is magnetic charge and therefore magnetic field lines begin nor end anywhere. The other two equations describe how the field circulate around their respective sources; the magnetic field circulates around electric current and time-varying electric field in Ampere's law with Maxwell's correction while the electric field circulates around time-varying magnetic fields in Faraday's law. The consequence of Maxwell's equations is since a changing electric field produces a magnetic field and vice versa, and the coupling between the two fields leads to the generation of electromagnetic waves. The microscopic equation has universal applicability for common calculation. They relate the electric and magnetic fields to total charge and total current, Including the complicated charges and currents in materials at the atomic scale. The version of Maxwell's equations based on the electric and magnetic scalar potential are preferred for explicitly solving equations as a boundary value problem, in analytic mechanic or for use in quantum mechanics. Maxwell's equations can be justified using a mathematical derivation.

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